Available online at www.sciencedirect.com
ScienceDirect
Computers, Environment and Urban Systems
31 (2007) 232–252
Computers, Environment and Urban Systems
www.elsevier.com/locate/compenvurbsys

Interactive evolutionary approaches to multiobjective spatial decision making: A synthetic review

Ningchuan Xiao [a,*], David A. Bennett [b], Marc P. Armstrong [b,c]

[a] Department of Geography, The Ohio State University, 1036 Derby Hall, 154 N Oval Mall, Columbus, OH 43210, United States
[b] Department of Geography, The University of Iowa, Iowa City, IA 52242, United States
[c] Program in Applied Mathematical and Computational Sciences, The University of Iowa, Iowa City, IA 52242, United States

Accepted in revised form 5 July 2006

Abstract

This paper reviews recent developments in evolutionary algorithms and visualization in the context of multiobjective spatial decision making. A synthetic perspective is employed to bridge these two areas and to create a unified conceptual framework that can be used to address a broad range of multiobjective spatial decision problems. In this framework, evolutionary algorithms are employed to generate optimal, or near-optimal, solutions to a problem being addressed. Alternatives created are then displayed in an interactive visual support system that can be used by decision makers to discover the competing nature of multiple objectives and to gain knowledge about the tradeoffs among alternatives.

Keywords: Multiobjective spatial decision making; Evolutionary algorithms; Interactive visualization

1. Introduction

Many geographical problems are not directly solvable through the straightforward application of a specific methodology. Such problems often require the participation of a

* Corresponding author. Tel.: +1 614 292 4072; fax: +1 614 292 6213.
E-mail address: xiao.37@osu.edu (N. Xiao).

0198-9715/$ - see front matter
doi:10.1016/j.compenvurbsys.2006.08.001

variety of stakeholders with different and often conflicting objectives. Locating a sanitary landfill, for example, may require decision makers to minimize its economic cost, and also to minimize negative environmental effects (Melachrinoudis, Min, & Wu, 1995). Or, a political redistricting plan may need to satisfy criteria such as equal population size of districts, compactness, and minority representation (Williams, 1995). In land use management, the incorporation of multiple objectives into decision making and the search for suitable land use policies are critical to sustainable regional development (Beinat & Nijkamp, 1998).

These and other types of multiobjective problems present a significant challenge to researchers for three main reasons. First, they are combinatorial optimization problems that often require a large amount of computation time to solve. Second, the search for solutions to these problems often involves the participation of stakeholders who have different backgrounds and view the problem from different perspectives. Finally, a solution that meets all criteria may not exist. Instead, stakeholders are required to examine tradeoffs among competing alternatives before a final solution can be reached. As a consequence, it is important to develop solution approaches that are (1) efficient in terms of their time complexity, (2) effective in terms of their ability to find a variety of high quality solutions, and (3) interactive so that decision makers can experiment with criteria, visually explore alternatives, and learn about a problem as they search for its solution.

This paper has three main goals. The first is to provide an overview of recent developments in multiobjective problem solving. We believe that borrowing ideas from other fields will benefit research on spatial decision making. The second is to present a new generalized conceptual framework that is intended to guide the design and implementation of evolutionary algorithms and visualization techniques. Evolutionary algorithms are particularly important because they can be used to solve multiobjective geographical problems efficiently, effectively, and often interactively. The third goal is to foster academic discussion about interactive spatial decision making using evolutionary algorithms and visual support systems.

The remainder of the paper is organized in five sections. We first discuss the background and scope of the paper in Section 2. Then, in Section 3, we discuss the theoretical and practical issues of using evolutionary algorithms for multiobjective optimization. In Section 4, we review methodological developments in evolutionary algorithms and visualization and place them in a conceptual framework that can be used to address multiobjective problems across a range of geographical applications. Two representative applications that have been published in the literature are reviewed in Section 5. We conclude the paper with a discussion of future research topics.

2. Multiobjective optimization: background

The first study of optimality in multiobjective problems is widely attributed to Pareto (1896). Here, we illustrate his analytical work, without loss of generality, using the following form of a multiobjective optimization problem:

$$\begin{array}{ll} \min & \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^{\mathrm{T}} \\ \text{subject to} & \mathbf{x} \in \mathbf{S} \end{array} \tag{1}$$

where $\mathbf{f}$ is a vector of m objective functions ($f_1, f_2, \ldots, f_m$) that are to be minimized, $\mathbf{x}$ is a vector of decision variables, and $\mathbf{S}$ is a set that defines all feasible solutions.

A solution to the above problem $\mathbf{x}'$ is said to dominate another solution $\mathbf{x}''$ if and only if $\forall i\, f_i(\mathbf{x}') \leqslant f_i(\mathbf{x}'')$ and $\exists i\, f_i(\mathbf{x}') < f_i(\mathbf{x}'')$. That is, if no objective function value of $\mathbf{x}'$ is greater (worse) than that of $\mathbf{x}''$, *and* there is at least one objective function value of $\mathbf{x}'$ that is less (better) than that of $\mathbf{x}''$, then we say that solution $\mathbf{x}'$ dominates solution $\mathbf{x}''$.

All feasible alternative solutions together form a solution space. When these solutions are placed in a space formed by the decision variables, it is called a decision space. Similarly, when solutions are placed in a space formed by the objectives, it is called an objective space. A subset of all feasible solutions is called the *non-dominated* or *Pareto optimal* set if its members are not dominated by any solution, and if solutions outside this subset are dominated by at least one solution in the subset. This set of non-dominated solutions is often called a *Pareto front*, as illustrated in Fig. 1.

Approaches to finding solutions to optimization problems can be placed into general categories. First, exact methods can be used to yield an optimal solution to a problem. The use of exact approaches may become impractical, however, when large size problems are addressed (Armstrong, 2000; Cooper, 1964; Garey & Johnson, 1979). To overcome this issue, researchers often resort to heuristic approaches that are more efficient. Heuristic methods, however, cannot guarantee that optimal solutions will be found, though the literature has demonstrated their effectiveness in finding optimal, or near-optimal solutions (Cooper, 1964; Reeves, 1993).

For multiobjective optimization problems, the dichotomy between exact and heuristic methods becomes more complicated. The ultimate goal of solving a multiobjective problem is to identify a Pareto optimal solution. To achieve this goal, three approaches have been developed in the literature (see Miettinen, 1999). The first approach, prior articulation of preferences, requires decision makers to reach a consensus about the relative importance (weight) of each objective *a priori*. The preferences of decision makers are then used in a technique called scalarization that converts a multiobjective problem into a single

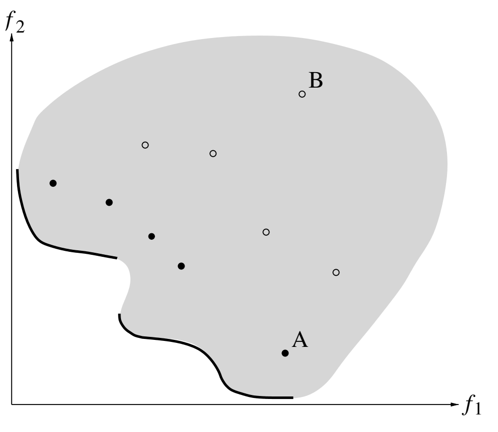

Fig. 1. Non-dominated solutions to a multiobjective optimization problem that minimizes two objectives (f_1 and f_2). The gray area represents a hypothetical objective space of the problem. The thick, black curves indicate the Pareto front. Circles represent selected solutions to the problem. Each solution marked as an open circle is dominated by at least one of the solutions marked as solid circles, while a solid circle is not dominated by any of the circles. All solutions represented as circles, however, are dominated by solutions on the Pareto front. Note that the Pareto front can be discontinuous.

objective problem which can be solved using a variety of solution approaches (Cohon, 1978). In many applications, however, it is difficult for decision makers to devise a satisfactory weighting scheme for scalarization because some features of a problem are not fully understood during the early stages of decision making. Moreover, it is often difficult for decision makers from different backgrounds to understand and quantify their own preferences because objectives may be formulated in complex mathematical forms that are difficult for non-analysts to understand. Indeed, some objectives may fall into cultural, religious, or other realms that will defy even heroic attempts to quantify them.

The second approach is based on an interactive (or progressive) articulation of preferences, in which the preferences of decision makers are refined and incorporated into the search process (Zionts & Wallenius, 1976). During each iteration of the process, decision makers are presented a (typically small) subset of non-dominated solutions, and based on these solutions they provide local information about their preferences for objectives. Then a single objective problem is formulated and solved. Solutions to this problem are used by decision makers to improve their understanding of the problem and to adjust their preferences, which, in turn, can be used to form a new problem. This process repeats until decision makers are satisfied. Though these methods can be used to encourage the participation of decision makers, there is no guarantee that an acceptable solution will be reached, and even if such a solution is identified, it cannot be guaranteed to be Pareto optimal.

Interactive decision making has become popular during the past several decades, and a large number of methods have been developed in the literature (see an overview by Miettinen, 1999, pp. 131–213). This is partly due to the more intensive involvement of decision makers in the process of searching for alternative solutions. This process can lead to more satisfactory final decisions (compared to prior approaches). However, interactive methods are often based on the generation of a small number of alternatives and they therefore may overlook important non-dominated solutions.

The third approach, called posterior articulation of preferences, does not require the intensive participation of decision makers during the process of generating alternatives. Instead, the application of this approach depends on methods that can be used to generate a diverse set of Pareto optimal solutions that are evenly distributed on the Pareto front; these solutions are subsequently presented to decision makers who make a final decision about the problem by examining and negotiating about the merits of alternatives. Two major difficulties have been identified as obstacles to the full application of this approach. First, it has been difficult to develop solution methods that can effectively generate the Pareto front. Traditionally, prior articulation methods have been used to generate the Pareto optimal solutions by systematically adjusting the associated parameters (e.g., preference weights) in order to yield different solutions (Brill, Chang, & Hopkins, 1982; Cohon, 1978). The problem with this approach, however, is that it may overlook important solutions, especially when the Pareto front contains concave, and/or discontinuous sections (Cohon, 1978; Deb, 2001; Ehrgott & Gandibleux, 2000; see also Fig. 1) . Second, the existence of a large number of non-dominated solutions will impose a substantial cognitive burden on decision makers who must somehow select a solution from the multitude of alternatives.

An important advantage of the posterior approach is also clear: a full representation of the Pareto front can present the true multiobjective structure of the problem, which may lead to a better decision (Brill, 1979). If an intuitive and user-friendly decision support tool is available, stakeholders can concentrate on examining and negotiating tradeoffs among interesting solutions (Jones, 1996; Lotov, Bushenkov, & Kameney, 2004). This approach may also foster a wider participation from stakeholders because they may be able to find "niche" solutions that are beneficial to their view of a problem and its resolution.

The last two decades have seen rapid development in two fields that shed new light on the use of posterior approaches. First, research has demonstrated the efficacy of evolutionary algorithms (EAs) as a new posterior approach to the generation of non-dominated solutions (see Coello Coello, 2000; Coello Coello, van Veldhuizen, & Lamont, 2002). EAs are heuristic methods that are more efficient than exact approaches and the literature has shown that EAs can be used to yield a diverse set of non-dominated solutions to approximate the Pareto front (Deb, 2001). The use of EAs in solving multiobjective spatial problems has also been documented (Bennett, Xiao, & Armstrong, 2004; Stewart, Janssen, & van Herwijnen, 2004; Xiao, Bennett, & Armstrong, 2002). A second new development is related to the emergence of spatial exploratory data analysis, especially using visualization techniques that enable users to examine unknown patterns in large spatial datasets (Anselin, 1998; Dykes, MacEachren, & Kraak, 2005; MacDougall, 1992; MacEachren & Kraak, 1997; Monmonier, 1989; Tukey, 1977). In the next two sections, we first review general issues that arise when EAs are applied to multiobjective optimization problems and then discuss how to incorporate EAs into a general framework that can be used for spatial decision making.

3. Multiobjective evolutionary algorithms

Evolutionary algorithms include a family of computer algorithms that share a set of common features. Different types of evolutionary algorithms include evolutionary strategies (Rechenberg, 1965), evolutionary programming (Fogel, 1962), genetic algorithms (Goldberg, 1989; Holland, 1975), and genetic programming (Koza, 1992). In general, these algorithms are based on a computer version of the Darwinian notion of natural selection and survival of the fittest. Different from traditional heuristic approaches that are problem-specific (see Cooper, 1964), evolutionary algorithms belong to a family of metaheuristic methods that can be applied to a wide range of optimization problems (Reeves, 1993). In addition to EAs, metaheuristic approaches generally include tabu search (Glover & Laguna, 1997) and simulated annealing (Kirkpatrick, Gelatt, & Vecchi, 1983).

The overall procedure of an EA is outlined in Fig. 2. In an EA, a solution to a problem being addressed is encoded as an individual. Encoding strategies vary among the different approaches. For example, a solution is encoded as a binary string in a genetic algorithm, while real or integer numbers are used in other types of EAs. An EA contains a number of individuals that collectively form a population of solutions. Initially, each individual is created randomly (Step 1 in Fig. 2). An evaluation process is then used to assign each individual a fitness value. Individuals that are better in terms of their objective function values are assigned a high fitness value, and those with high fitness values are more likely to be selected to create individuals in the next generation.

A new generation of individuals is created either by copying (fit) members without alteration or by modifying them in an attempt to increase their fitness. A typical operator that is used to modify members is called recombination, which combines information from two selected parent individuals to create a new solution. The main search power of recombination operations is based on the exploitation of existing (best) solutions. Searching for

```
Procedure EA
Initialize the population
Evaluate initial population
Repeat
    Conduct selection
    Apply evolutionary operators to generate new solutions
    Evaluate solutions in the population
Until termination criteria are met
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Fig. 2. A general EA procedure.

optimal solutions, however, also needs to explore those parts of a solution space that have not been represented in current solutions (Eiben & Schippers, 1998; Michalewicz, 1996). Mutation operations accomplish this task by randomly modifying a (typically small) portion of individuals in the current generation to yield new solutions. The new generation of individuals is then re-evaluated using a fitness function and the processes of recombination and mutation repeats until predefined termination rules (e.g., the total number of generations) are met.

A selection operation plays a critical role in an EA because it "encourages" individuals with high fitness value to evolve toward an optimum. Multiobjective optimization requires a special fitness assignment algorithm that accounts for the (non)domination status of each individual. A widely used approach is called Pareto ranking (Goldberg, 1989), while other methods are also available (see, for example, Fonseca & Fleming, 1993). The Pareto ranking method sorts the population and assigns each individual a rank that indicates its non-domination status. All non-dominated individuals in the current population are assigned a rank value of 1. The remaining un-ranked individuals are re-evaluated and those that are now non-dominated will be assigned a rank value of 2. This process continues until all individuals are ranked. For example, let us assume that the current population consists of the individuals represented as open and solid circles in Fig. 1. In this population, solid circle solutions will be assigned the rank of 1, unmarked open circle solutions are ranked 2, and solution B is ranked 3.

A number of methods have been developed to convert Pareto ranks into fitness values, with the intention of diversifying a population (Fonseca & Fleming, 1993; Horn, Nafpliotis, & Goldberg, 1994; Srinivas & Deb, 1995; van Veldhuizen, 1999; Zitzler & Thiele, 1999). Diversity in this case refers to the variation among individuals in a population (Langdon & Poli, 2002, p. 248). Diversity is useful in EAs because a diverse population can be used to prevent a few "good" solutions from dominating the entire population, especially during early iterations when these good solutions are normally far from optimal. Such diversity is particularly important if an EA is to be applied successfully to a multiobjective optimization problem (Deb, 2001). Here, we describe the approach developed by Srinivas and Deb (1995), though detailed discussion of other methods also can be found (see Coello Coello, Van Veldhuizen, & Lamont, 2002; Deb, 2001). This technique first assigns a fitness to each individual solution according to its rank. For example, solutions represented by solid circles in Fig. 1 will be assigned the highest fitness value. Then, the fitness value of each individual is modified by a niche count, measured by the number of individuals in its neighborhood. The meaning of neighborhood is based on a distance measure defined in the objective space (e.g., Fig. 1) or decision space (e.g., the spatial similarity between two alternatives). For a particular domination rank, a solution that is located in a less crowded area of the objective or decision space (e.g., *A* in Fig. 1) will have a higher fitness value than those in a crowded area (e.g., other solid circles in Fig. 1).

Recently, Xiao and Armstrong (2003) extended the ideas of population specialization (Bennett, Armstrong, & Wade, 1996) and the island model of EAs (Martin, Lienig, & Cohoon, 1997) and developed a specialized island model (SIM) that can be used to effectively diversify an EA population and thus find a greater number of non-dominated solutions. In SIM, the entire population is divided into several subpopulations, each of which performs a complete set of evolutionary operations. A subpopulation is not required to search for solutions with respect to all objectives, however. Instead, some subpopulations are *specialized* to search for solutions with respect to a subset of original objectives. For example, for a problem to minimize three objective functions (i.e., $\min[f_1,f_2,f_3]^{\mathrm{T}}$), a subpopulation can be specialized in the first two objectives (i.e., its objective functions are $\min[f_1,f_2]^{\mathrm{T}}$), while all constraints of the original problem remain the same. A mechanism called migration is used to send a number of individuals among subpopulations to increase the overall diversity of each subpopulation by introducing "exotic" individuals from other subpopulations. Applications of this approach have demonstrated that it can be used to find a higher proportion of non-dominated solutions (see Armstrong, Xiao, & Bennett, 2003; Bennett et al., 2004; Xiao & Armstrong, 2005).

Finally, we note that other metaheuristic approaches (e.g., tabu search and simulated annealing) have also been used to address multiobjective optimization problems. These approaches are developed based on a neighborhood concept. For an existing solution to a problem, its neighborhood is formed by solutions that are considered to be close in the solution space (Glover & Laguna, 1997; Kirkpatrick et al., 1983). The neighbors of a solution can be obtained by adjusting a portion of that solution (see Anderson, 1996). In these metaheuristic methods, an initial solution is improved by iteratively moving it to another (mostly better) solution in its neighborhood. This approach is similar to prior methods in which the multiple objectives are converted to a single objective (see Ehrgott & Gandibleux, 2000). Recent developments have been focused on approximating the Pareto front in a more systematic manner (Ben Abdelaziz, Chaouachi, & Krichen, 1999; Czyzzak & Jaszkiewicz, 1998; Duh & Brown, this volume; Gandibleux, Mezdaoui, & Fréville, 1997). This new strategy normally includes two phases. In the first phase, the algorithm maintains an individual solution that tends to move to a dominating neighbor solution until no such solution can be found in the neighbors. In the second phase, the algorithm encourages the individual solution to move to non-dominated neighbor solutions. The second phase stops when no non-dominated solution can be found in the neighborhood. Though these approaches are effective for some particular cases, they have not been fully tested using benchmark multiobjective optimization problems (see Deb, 1999), and their application in spatial problems has been limited. Consequently, in this paper, we focus on EAs as a posterior means of solving multiobjective spatial decision problems.

4. A conceptual framework for multiobjective spatial decision making

A general, theoretical framework for decision making can be constructed based on the pragmatic approaches that have been established by philosophers who were concerned with a search for solutions to problems. Dewey (1910), for example, has argued that three steps are typically followed when decisions are made: what is the problem, what are the alternatives, and which alternative is best? Simon (1960) concurred with the three main steps in decision making, though he used different names: intelligence, design, and choice. These three steps have been adopted and extended in a variety of decision support frameworks (Brightman, 1978; Densham, 1991; Cameron & Abel, 1996; Jankowski & Nyerges, 2001). In this paper, we base our conceptual framework on Simon's traditional three steps, which are more closely tied to the distinction between design (searching for optimal solutions) and application (determining a suitable final solution). More specifically, our conceptual framework for multiobjective spatial decision making is created from the perspective of evolutionary algorithms and exploratory data visualization techniques. Fig. 3 illustrates the fundamental elements of this framework, as well as the three steps suggested by Simon (1960). In this section we discuss two major components of this framework. We first discuss issues in the design and implementation of EAs for alternative generation. Hybridization between EAs and other approaches will also be discussed below. We then discuss the role of visualization techniques in helping decision makers understand problem structures and tradeoffs among alternatives. The first component of this framework (i.e., problem formulation) is not a particular focus of this paper, and we discuss it in a problem-specific manner in Section 5.

4.1. Generation of alternatives using evolutionary algorithms

In the context of evolutionary algorithms, problems are formulated differently than they are in traditional linear programming models, though the general mathematical form of a problem still holds (Eq. (1)). For an EA, a problem is more directly formulated in an algorithmic manner: an appropriate data structure is designed to encode solutions, and evolutionary operations are specified to handle constraints. The fitness evaluation techniques discussed in the previous section can be generally applied to a broad range

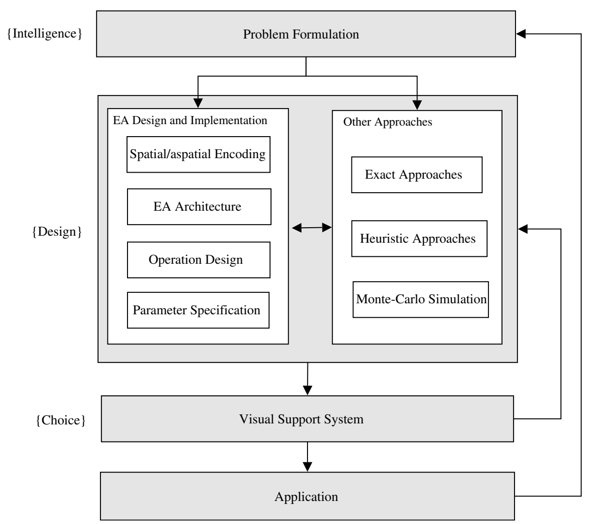

Fig. 3. A conceptual framework of using evolutionary algorithms for multiobjective spatial decision making.

of problems. To address spatial multiobjective problems, we focus on representation strategies and the implementation of evolutionary operations.

Hosage and Goodchild (1986) first used a genetic algorithm to address the p-median problem, with a goal to locate p facilities in a spatial network of n nodes such that the total distance between each node and its closest facility is minimized. In their approach, a binary string with a length of n was used to represent a solution to the problem. If the number of bits with a value of 1 in a string is greater than p, its original fitness value was decreased using a penalty function. Their results, unfortunately, tended to be trapped in local optima even for small problems. Later, researchers (Bianchi & Church, 1993; Dibble & Densham, 1993) used a string of p integers to represent the location of the p facilities in a solution, and "extremely good, if not optimal, solutions" were found (Church & Sorensen, 1996). Though recent developments on the same subject have demonstrated that improved performance can also be gained through the design of new operations (Bozkaya, Zhang, & Erkut, 2002; Estivill-Castro & Torres-Velázquez, 1999; Jaramillo, Bhadury, & Batta, 2002), the use of integer strings to represent spatial problems has become a common choice in this area of EA research.

For spatial problems that are subject to more complicated spatial constraints (such as contiguity), however, modifications to the above representation strategy is needed. Krzanowski and Raper (1999), for example, addressed the problem of locating transmitters for wireless networks using a genetic algorithm, in which the location and radius of a transmitter are represented as a tuple of (x, y, R). A string of such tuples is used to encode a solution to the problem. To search for an optimal set of contiguous cells in a raster map, Brookes (2001) designed a genetic algorithm in which each individual consists of a number of parameters that are used by a region-growing algorithm (Brookes, 1997) to create a site. These approaches, though effective, are *ad hoc* because they can only be applied to a particular type of problem. In an attempt to develop a more general framework that can be used for a broad range of geographical problems, Xiao et al. (2002) and Xiao (2006) designed a representation strategy based on graph theory. In this approach, a string of integers is used to indicate spatial units. Moreover, the spatial connectivity of each spatial unit is maintained. Based on this representation, evolutionary operations can be designed to satisfy spatial constraints.

A variety of evolutionary operations (recombination and mutation) have been developed in the literature, each of which is particular to the encoding strategy used, though some common traits can still be observed. The design of recombination operations is often tightly related to the specification of spatial constraints, a critical issue in evolutionary algorithm research (Deb, 2001; Michalewicz, 1996). In general, two major approaches can be identified. The first type exhibits "evolutionary" characteristics because it relies on a penalty function to discourage the promotion of infeasible solutions to subsequent generations. Though this approach has been successful in many numerical optimization applications (Michalewicz, 1996; Xiao & Armstrong, 2005), it has not been fully tested for spatial problems; the experiments by Hosage and Goodchild (1986), for example, were unsuccessful.

The second type of approach to constraint handling is based on the development of specific evolutionary operations (in combination with encoding strategies) to ensure that (1) only feasible solutions are encoded, and (2) only feasible solutions may result from evolutionary operations. A typical example of this approach is the "greedy crossover" developed by Grefenstette, Gopal, Rosmaita, and van Gucht (1985) to solve the traveling salesman problem using a genetic algorithm in which the crossover operation only creates a feasible solution (a tour that traverses every location in an area without cycling). This type of approach is adopted for most spatial optimization problems. For example, to solve the *p*-median problem using evolutionary algorithms, traditional single- or multi-point crossover mechanisms have been employed. However, these operations are often modified to ensure that the results of a recombination will yield a feasible solution (Bennett et al., 2004; Bozkaya et al., 2002; Dibble & Densham, 1993).

When spatial constraints are required, more sophisticated operations may be needed. For example, to solve site search problems, a site is formed by a contiguous set of selected land parcels, meaning that one can move from one selected parcel to another without leaving the site. To maintain the contiguity of a solution, Xiao (2006) developed an operation that locally adjusts the shape of a site by removing a land parcel from and then adding a new land parcel to the site. Both removal and addition processes can only be performed if doing so will not create an infeasible (non-contiguous) site. This operation is called "local search" by Xiao (2006), though it should be noted that such an operation can be considered as an asexual crossover or transposition that modifies a single solution to create a new one (see Simões & Costa, 2000). This operation is different from a mutation and is conducted based on the fitness function value of a particular solution.

Mutation operations are used to help the overall search escape local optimal traps. The most commonly used mutation operations in spatial problems are similar to the ones in traditional genetic algorithms, which alter a small portion of an encoding to create a new solution. Unlike recombination operations, mutation is only occasionally conducted because a mutation may disturb current individual solutions by introducing harmful "noise". For example, a typical EA would allow a small portion (e.g., about 1 percent) of the individuals in a population to experience a mutation operation (Michalewicz, 1996). However, the occasional use of mutation makes the overall search more exploratory, meaning that new genetic materials are introduced into the current population of solutions. Studies have shown that an EA cannot return high-quality solutions when mutation operations are not implemented (see Goldberg, 1989; Xiao, 2006).

Finally, we note that while the focus of this paper is placed on evolutionary algorithms, other solution approaches can be incorporated into EAs to improve their performance. For example, Bennett et al. (2004) used results from integer programming to form a subset of initial solutions for an EA. Their results suggest that this strategy can expedite the convergence of the EA. In many applications, a Monte Carlo approach that randomly generates alternative solutions can also be useful because these random solutions can be used to reveal the general scope of the solution space (see Armstrong et al., 2003). The general idea of incorporating other methods (including heuristics) into EAs follows the trend of hybridization between evolutionary algorithms and other local search algorithms (Anderson, 1996; Preux & Talbi, 1999; Voss, Martello, Osman, & Roucairol, 1999). Previous research has shown that the use of hybridization strategies in EAs can greatly improve EA performance (Estivill-Castro & Murray, 2000a, 2000b; Krzanowski & Raper, 1999; Lin, Hwang, & Wang, 2001; Ruiz-Andino, Araujo, Sáenz, & Ruz, 2000; Zhang & Armstrong, 2005).

4.2. Visual support tools

The use of visualization techniques has long been considered a critical component for optimization in general (Jones, 1996; Lotov et al., 2004), and spatial decision support systems in particular (Armstrong, Densham, Lolonis, & Rushton, 1992; Armstrong & Lolonis, 1989; Densham, 1994; Jankowski, Andrienko, & Andrienko, 2001; Malczewski, 1999; Malczewski, Pazner, & Zaliwska, 1997). In the context of spatial decision making, the connection between a solution and its objective function values must be made so that decision makers can examine alternatives interactively. To achieve this goal, it is necessary to understand the relationship between the decision space and the objective (or criteria) space (Church, Loban, & Lombard, 1992; Schilling, ReVelle, & Cohon, 1983).

For many numerical optimization problems, plotting solutions under consideration is an intuitive way to provide context. When decisions have a geographical component, however, it is also intuitive to adopt cartographic and other visualization techniques to present alternatives (Armstrong et al., 1992; Malczewski et al., 1997). Recently, Bennett et al. (2004) used the term geographical space to refer to the set of all possible spatial patterns (represented as maps) of alternative solutions.

Since decision and objective spaces may have a high dimensionality, a set of visualization techniques has been developed to convert high dimensional information into forms that humans can easily examine. In particular, scatterplot matrices and parallel coordinates are often used (Anselin, 1998; Buja & Cook, 1996; Swayne, Cook, & Buja, 1998). In a scatterplot matrix, the number of rows (or columns) equals the number of dimensions of the information displayed. Four variables, for example, require a 4 by 4 matrix, in which a non-diagonal cell at the *i*th row and the *j*th column contains a plot with the two axes formed by the *i*th and *j*th variables. Scatterplot matrices are effective in projecting high dimensional data into two dimensional space, and they are commonly used in many statistical software packages. However, it is difficult for users to gain knowledge about the overall relationships that are exhibited among alternatives with respect to all objectives.

An alternative to the scatterplot matrix approach is parallel coordinates (Buja & Cook, 1996; Inselberg, 1981). To display *n*-dimensional information, *n* vertical lines are drawn; each vertical line is an axis that represents a dimension of the information. Fig. 4 illustrates the use of this technique in a multiobjective optimization context, where the space of six objectives is visualized using six parallel coordinates and each vertical line represents an objective. The objective function values of a solution are linked to form a line, which is called a value path (Melachrinoudis et al., 1995; Schilling et al., 1983). Suppose the overall objective is to minimize the six objectives. In Fig. 4, the solution corresponding to value path *a* is dominated by all other solutions, the solution corresponding to value path *d* is

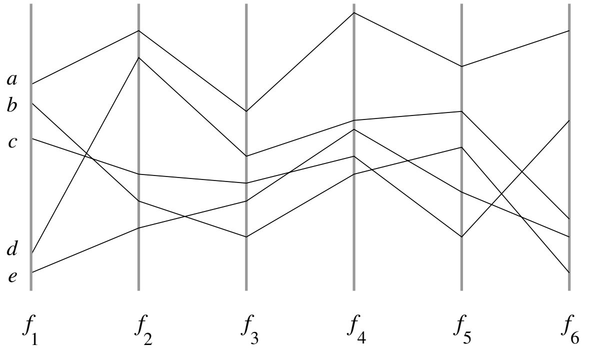

Fig. 4. A parallel coordinate plot with six dimensions.

dominated by *e*, and solutions corresponding to other value paths (*b*, *c*, and *e*) are nondominated ones. The disadvantage of parallel coordinates is that a different ordering of the coordinates may yield different visual impressions.

To fully support interactive decision making, the displays described above (maps, parallel coordinates, and scatterplot matrix) must be linked so that when a decision maker selects a certain value path, for example, the system should simultaneously display the selected solution in the scatterplot matrix and map. This technique, called brushing, or more specifically in our context, geographical brushing, has been used in a variety of visualization applications (Dang, North, & Shneiderman, 2001; Gahegan, Takatsuka, Wheeler, & Hardisty, 2002; Monmonier, 1989). A more general approach to linking multiple views is demonstrated by Carr (2001).

In addition to linking visual displays together, another key feature in a user-friendly visual support system is the ability to allow users to search for similar or dissimilar alternatives. This feature is particularly important when a large number of alternatives are created and presented to decision makers. This approach can be observed in our daily life experiences. For example, Amazon.com will automatically (with classification algorithms running behind the scene) present similar book titles to a customer who is viewing a certain title; libraries often display books in a same category together on a shelf. For multiobjective spatial decision problems, we identify four types of similarity/dissimilarity among any two alternatives (see also Fig. 5):

- Type I. The two alternatives that are similar in objective space are also similar in geographical (or decision) space.
- Type II. Two solutions are different in objective space and are also different in geographical space.
- Type III. Two similar solutions in objective space are different in geographical space.
- Type IV. Two alternatives that are similar in geographical space are different in objective space.

Types I and II represent "normal" cases for many solutions because given two solutions that are close (far away) in an objective space, a user would expect them to be close (far away) in a geographical space. Types III and IV, however, are more unusual because they represent circumstances when two alternatives are close in one space but different in the other. Though these types may not be common in our analogy of online shopping or library browsing, they are important for multiobjective spatial decision making because

		Geographical Space	
		Similar	Dissimilar
Objective Space	Similar	Type I	Type III
	Dissimilar	Type IV	Type II

Fig. 5. Four types of similarity and dissimilarity of alternatives.

alternatives with a similar spatial pattern (in geographical space) but different objective function values (in objective space), or vice versa, can be presented to decision makers so that tradeoffs among them can be fully discussed (Bennett et al., 2004).

The above discussion raises the important issue of measuring similarity between alternatives. Bennett et al. (2004) explored measures that define the similarity of two alternatives in objective space using Euclidean distance in a multi-dimensional space formed by objectives. The similarity (or distance) between two alternatives in terms of their geographical patterns is calculated as the summation of a binary variable t_j over all spatial units. The value of t_j is 1 if the jth spatial unit is assigned to a same land use type for both alternatives. Xiao and Armstrong (2005) have suggested that this type of measure can be weighted by the area of spatial units.

The techniques discussed above can be used as the core components of a visual support system for multiobjective spatial decision problems, though we note that additional visualization tools (such as color rendering and animation) can also play a significant role. A prototype of such a system is illustrated in Fig. 6. Though basic features of this prototype

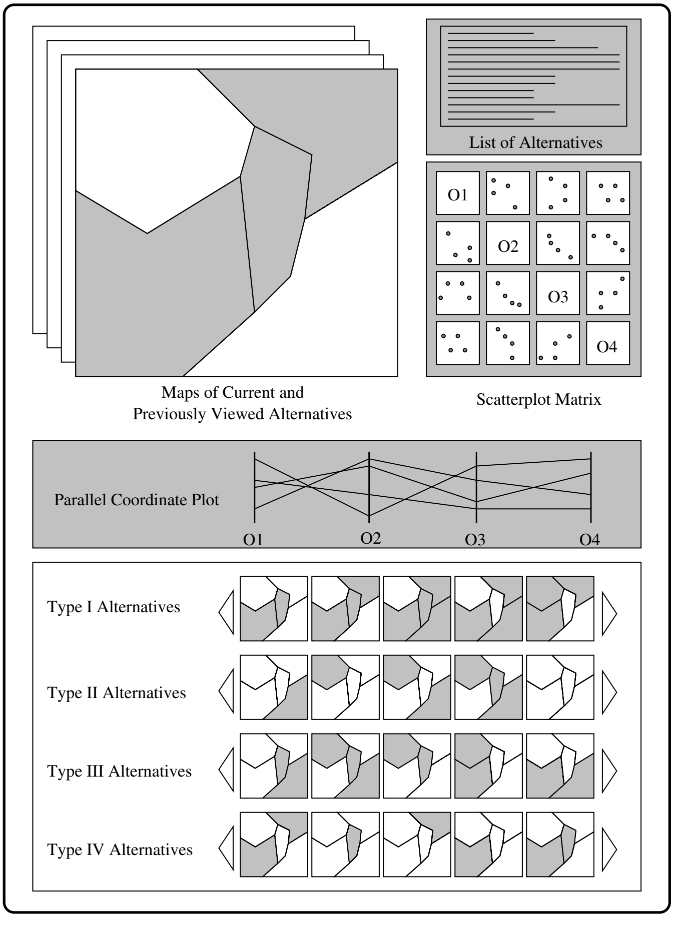

Fig. 6. A prototype of a visual support system for multiobjective spatial decision.

can be found in visual systems reported in the literature (see, for example Andrienko & Andrienko, 1999; Gahegan et al., 2002; Jankowski et al., 2001; Xiao & Armstrong, 2006), some unique design features should be noted. First, the visual system must store the recently viewed alternatives and their maps so that decision makers can switch back and forth for comparison (similar to the trace function for group decision support systems suggested by Armstrong (1993)). We use a metaphor of the map stack (see Bruns & Egenhofer, 1997) for this purpose in Fig. 6, though other visualization techniques such as the perspective wall (Mackinlay, Robertson, & Card, 1991) or translucent window (Packard, 2000) are also useful. Second, four series of maps are arranged horizontally at the bottom of the prototype; each series represents the alternatives that belong to one of the four similarity/dissimilarity types related to the alternative currently viewed in the main map window in the upper-left corner. Maps arranged in this way are called small multiples, a design concept that is an effective visualization tool for data exploration (Tufte, 1990). When a user changes the current alternative, its similar/dissimilar alternatives can be dynamically queried from the database that contains all non-dominated alternatives and then displayed (Ahlberg, Williamson, & Shneiderman, 1992).

5. Review of applications

Evolutionary approaches have been applied to a variety of spatial decision problems such as land use and environmental policy (Bennett et al., 2004; Stewart et al., 2004), land acquisition (Aerts, van Herwijhen, Janssen, & Stewart, 2005; Xiao et al., 2002), routing (Guimarães Pereira, 1996; Zhang & Armstrong, 2005), and urban and regional planning (Balling, Taber, Brown, & Day, 1999; Feng & Lin, 1999). Here, we focus on two representative examples that demonstrate the utility of EAs during interactive geographical problem solving and the application of the interactive approach discussed in this paper. The applications are based on a Pareto search for solutions to multiobjective problems (i.e., scalarization techniques are not used).

Feng and Lin (1999) developed a genetic algorithm that can be used to generate Pareto optimal alternative sketch maps for urban planning. In their context, a sketch map serves as a guideline when a new town is planned for construction in an undeveloped area. A sketch map, often in a coarse spatial resolution, specifies possible configurations of land use (e.g., residence, commerce, and industry) and transport networks for each spatial unit (cells). Based on a sketch map, a more detailed layout, called a development map, will be subsequently created by developers and planners. In their application, two objectives are defined. The first is to maximize environmental harmony, and the second is to maximize development efficiency. A feasible solution to this problem must satisfy three constraints: each cell must be assigned a land use and links to a transport network, all land uses must be assigned to the cells in a planned area, and at least one transport path must exist between any two cells in the area.

A binary encoding strategy was used by Feng and Lin (1999) to represent a feasible solution. A string of $3 \times I \times J + E$ bits are used, where I and J are, respectively, the number of columns and rows, and E is the number of potential connections for each cell. A cell may be connected to four neighbors (up, down, left, and right). The entire string is organized as follows:

$$a_{11}b_{11}c_{11}a_{21}b_{21}c_{21}\cdots a_{ij}b_{ij}c_{ij}\cdots a_{IJ}b_{IJ}c_{IJ}d_1\cdots d_E,$$

where a_{ij}, b_{ij}, c_{ij}, and d_e are binary variables. The land use assigned to the cell at column i and row j is one of the eight values calculated as $a_{ij}2^0 + b_{ij}2^1 + c_{ij}2^2$. In this application, d_e is 1 if the eth potential link is used in the plan.

Feng and Lin (1999) designed a set of evolutionary operations to search for optimal solutions, while maintaining feasibility. The feasibility of a solution can be easily checked. During the initialization step, the operation is repeated until a specified number of initial feasible solutions is created. The recombination operation creates two new individuals by exchanging two bits that are randomly picked from two parent solutions. A mutation operation is used to randomly reverse a bit in a solution. Infeasible new individuals are ignored and the processes of recombination and mutation repeat until the required number of new individuals are created. The fitness value of each individual is then calculated using a method similar to the Pareto-rank approach described above. Feng and Lin (1999) applied their genetic algorithm to a case study in a new town, Tanhai, Taiwan. The planned area contains 40 cells, each of which has an area of approximately 100 ha. They identified four non-dominated solutions and all are better (in both objectives) than the official sketch layout.

Our second example application is related to environmental policies in a study area located in Cache River watershed in southern Illinois (Bennett et al., 2004). In this area, environmental policies such as the conservation reserve program (CRP) play an important role in restoring original swamps that are considered to exhibit high environmental benefits such as reducing non-point pollution. In this study area, three major stakeholders with different objectives were identified. Government agencies (the United States Department of Agriculture, or USDA) wish to maximize environment benefit; local farmers seek to maximize income; and the general public are interested in minimizing public investment. Each of these objectives can be quantitatively defined.

Bennett et al. (2004) developed an EA to search for landscapes that are of interest to different stakeholders. Here, a landscape consists of 961 farm fields. A farm field can be assigned to one of 16 land use types (corn, soybean, wheat, hay, double crop, and 11 other CRP cover types ranging from grass to wetland restoration). In addition, USDA guidelines require that only 25 percent area of a region can be enrolled in the CRP. In their EA, an individual solution is encoded as a string of 961 integers, where the value of each integer ranges from 0 to 15. The recombination operation is based on a single-point crossover. Three mutation operations were designed to randomly change a portion of the farm fields from one land use type to another. If an individual solution has more than 25 percent of its overall area enrolled in the CRP, a repair mechanism is used to randomly switch farm fields from CRP types to a crop type. Individuals are evaluated using an approach similar to the niching and sharing method described above.

Bennett et al. (2004) illustrated the efficacy of their EA in finding optimal or near-optimal solutions by comparing the EA outcome with results from an integer programming model. The integer programming approach is based on a series of weighting schemes that represent different stakeholder preferences. For many schemes, however, the integer programming model could not return feasible solutions.

Finally, Bennett et al. (2004) developed an interactive visualization tool that provides several of the functions illustrated in Fig. 6. This tool includes a list of alternatives and a scatterplot of the objective space. To help users identify interesting alternatives, similarity between alternatives is measured in both objective and decision spaces. An alternative can be selected by highlighting a point in the scatterplot or in the list, and a map of the selected alternative can be drawn subsequently. Points in the scatterplot can be rendered using a gray scale where alternatives with similar spatial pattern or objective function values to the currently selected one are displayed in a dark gray. Using this system, a user can examine the tradeoff among alternatives by comparing their objective function values and visualizing their spatial configurations using maps.

6. Discussion and conclusions

Researchers have developed three major alternative generating approaches with respect to how preferences are articulated: prior, interactive, and posterior. The research reviewed in this paper comprises an interactive evolutionary approach that does not necessarily fall into a particular category. Instead, it represents a new paradigm of multiobjective problem solving, as described by Casti (1997, p. 171): "you don't solve it, you evolve it."

The generation of high-quality alternatives is a key to the success of multiobjective spatial decision making. The methodological essence of evolutionary algorithms is based on a process of evolution from initially random individual solutions toward a diverse set of optimal, or near-optimal solutions. Because EAs are population based, it is possible to design algorithms to encourage the emergence of diversity and optimality. Though we note that other heuristic methods (such as simulated annealing) share the same evolutionary spirit, evolutionary algorithms are particularly appropriate for multiobjective decision making.

The use of visual support systems in the context of multiobjective spatial decision making is also consistent with the evolutionary perspective. The process of exploring alternatives is one of abduction, or "inference to the best explanation" (Josephson & Josephson, 1994, p. 5), and is consistent with the views espoused by Simon (1960) and Dewey (1910). The visual techniques and framework reviewed in the paper can be used to help decision makers discover the competing nature of different objectives and the tradeoffs among alternatives. Substantial knowledge about the problem can evolve via experiments using the visual system.

To fully benefit from the conceptual framework set forth in this paper, however, a number of critical issues require further investigation. Here, we identify three important future research topics. We believe that by addressing these issues, decision makers will be equipped with more effective tools to solve multiobjective spatial decision problems.

1. A graph theory based representation strategy is a "natural" way to encode solutions to spatial optimization problems in an EA. Though empirical studies have demonstrated the effectiveness of this strategy in finding optimal or near-optimal solutions, a theoretical analysis of convergence using this encoding strategy is still needed. Previous research on other representation strategies (especially binary strings, see Goldberg, 1989 & Nix & Vose, 1992) can be used as an example for such theoretical work.
2. Spatial optimization problems often have constraints that are difficult to translate into mathematical forms. Though evolutionary algorithms have proven to be effective in addressing such constraints, a unified approach to constraint handling for a wide range of spatial problems would be useful.
3. The evolutionary algorithms and visualization techniques discussed in this paper have been separately implemented in different forms, and efforts are needed to integrate them into a more coherent system that can be used to address spatial applications. Xiao and

Armstrong (2006) have demonstrated the use of object oriented programming techniques to implement an EA that can be applied in other cases. EA results can be stored in computer files that will be subsequently loaded into a visual support system. Though such a file level integration is sufficient for some application cases, it may not be suitable for applications that involve stakeholders from a variety of backgrounds with different technical skills. Given the increasing availability of geographical information systems (GIS), integrating multiobjective spatial decision tools into GIS and other visualization systems is a fruitful direction for future work.

References

Aerts, J. C. J. H., van Herwijhen, M., Janssen, R., & Stewart, T. J. (2005). Evaluating spatial design techniques for solving land-use allocation problems. *Journal of Environmental Planning and Management, 48*(1), 121–142.

Ahlberg, C., Williamson, C., & Shneiderman, B. (1992). Dynamic queries for information exploration: an implementation and evaluation. In *Human factors in computer systems CHI '92 conference proceedings* (pp. 619–626). Reading, MA: Addison-Wesley.

Anderson, E. J. (1996). Mechanisms for local search. *European Journal of Operational Research, 88*, 139–151.

Andrienko, G. L., & Andrienko, N. V. (1999). Interactive maps for visual data exploration. *International Journal of Geographical Information Science, 13*(4), 355–374.

Anselin, L. (1998). Exploratory spatial data analysis in a geocomputational environment. In P. A. Longley, S. M. Brooks, R. McDonnell, & B. Macmillan (Eds.), *GeoComputation: A primer* (pp. 77–94). Chichester, England: John Wiley & Sons.

Armstrong, M. P. (1993). Perspectives on the development of group decision support systems for locational problem-solving. *Geographical Systems, 1*(1), 69–81.

Armstrong, M. P. (2000). Geography and computational science. *Annals of the Association of American Geographers, 90*(1), 146–156.

Armstrong, M. P., Densham, P. J., Lolonis, P., & Rushton, G. (1992). Cartographic displays to support locational decision making. *Cartography and Geographic Information Systems, 19*(2), 154–164.

Armstrong, M. P., & Lolonis, P. (1989). Interactive analytical displays for spatial decision support systems. In *Auto-Carto 9* (pp. 171–180). Baltimore, MD: ASPRS, ACSM.

Armstrong, M. P., Xiao, N., & Bennett, D. A. (2003). Using genetic algorithms to create multicriteria class intervals for choropleth maps. *Annals of the Association of American Geographers, 93*(3), 595–623.

Balling, R. J., Taber, J. T., Brown, M. R., & Day, K. (1999). Multiobjective urban planning using genetic algorithm. *Journal or Urban Planning and Development, 125*(2), 86–99.

Beinat, E., & Nijkamp, P. (1998). Land-use management and the path towards sustainability. In E. Beinat & P. Nijkamp (Eds.), *Multicriteria analysis for land-use management* (pp. 1–13). Dordrecht: Kluwer Academic Publishers.

Ben Abdelaziz, F., Chaouachi, J., & Krichen, S. (1999). A hybrid heuristic for multiobjective knapsack problems. In S. Voss, S. Martello, I. Osman, & C. Roucairol (Eds.), *Meta-heuristics: Advances and trends in local search paradigms for optimization* (pp. 205–212). Dordrecht: Kluwer Publishers.

Bennett, D.A., Armstrong, M.P., Wade, G.A. (1996). Agent mediated consensus-building for environmental problems: a genetic algorithm approach. In *Third international conference/workshop on integrating GIS and environmental modeling, Santa Fe, NM*. National Center for Geographic Information and Analysis. CD and http://www.ncgia.ucsb.edu/.

Bennett, D. A., Xiao, N., & Armstrong, M. P. (2004). Exploring the geographic ramifications of environmental policy using evolutionary algorithms. *Annals of the Association of American Geographers, 94*(4), 827–847.

Bianchi, G. & Church, R. (1993). A non-binary encoded GA for a facility location problem. Working paper, Department of Geography, University of California, Santa Barbara.

Bozkaya, B., Zhang, J., & Erkut, E. (2002). An efficient genetic algorithm for the *p*-median problem. In Z. Drezner & H. W. Hamacher (Eds.), *Facility location: Applications and theory* (pp. 179–205). Berlin: Springer.

Brightman, H. (1978). Differences in ill-structured problem solving along the organizational hierarchy. *Decision Sciences, 9*(8), 1–18.

Brill, E. D. Jr., (1979). The use of optimization models in public-sector planning. *Management Science, 25*(5), 413–422.

Brill, E. D., Jr., Chang, S.-Y., & Hopkins, L. D. (1982). Modeling to generate alternatives: the HSJ approach and an illustration using a problem in land use planning. *Management Science, 28*(3), 221–235.

Brookes, C. J. (1997). A parameterized region-growing programme for site allocation on raster suitability maps. *International Journal of Geographical Information Science, 11*(4), 375–396.

Brookes, C. J. (2001). A genetic algorithm for designing optimal patch configurations in GIS. *International Journal of Geographical Information Science, 15*(6), 539–559.

Bruns, H. T., & Egenhofer, M. J. (1997). User interfaces for map algebra. *Journal of the Urban and Regional Information Systems Association, 9*(1), 44–54.

Buja, A., & Cook, D. (1996). Interactive high-dimensional data visualization. *Journal of Computational and Graphical Statistics, 5*(1), 78–99.

Cameron, M. A., & Abel, D. J. (1996). A problem model for spatial decision support systems. In M. J. Kraak & M. Molenaar (Eds.), *Advances in GIS Research II. Proceedings of the 7th international symposium on spatial data handling* (Vol. I, pp. 3A.25–3A.36). Delft: The Netherlands.

Carr, D. B. (2001). Designing linked micromap plots for states with many counties. *Statistics in Medicine, 20*, 1331–1339.

Casti, J. L. (1997). *Would-be worlds: how simulation is changing the frontiers of science*. New York: John Wiley & Sons.

Church, R. L., Loban, S. R., & Lombard, K. (1992). An interface for exploring spatial alternatives for a corridor location problem. *Computers & Geosciences, 18*(8), 1095–1105.

Church, R. L., & Sorensen, P. (1996). Integrating normative location models into GIS: problems and prospects with the p-median model. In P. Longley & M. Batty (Eds.), *Spatial analysis: Modelling in a GIS environment* (pp. 167–183). Longman Geoinformation.

Coello Coello, C. A. (2000). An updated survey of GA-based multiobjective optimization techniques. *ACM Computing Surveys, 32*(2), 109–143.

Coello Coello, C. A., van Veldhuizen, D. A., & Lamont, G. B. (2002). *Evolutionary algorithms for solving multi-objective problems*. New York: Kluwer Academic Publishers.

Coello Coello, C. A., Van Veldhuizen, D. A., & Lamont, G. B. (2002). *Evolutionary algorithms for solving multi-objective problems*. New York: Kluwer Academic.

Cohon, J. L. (1978). *Multiobjective programming and planning*. New York: Academic Press.

Cooper, L. (1964). Heuristic methods for location-allocation problems. *SIAM Review, 6*, 37–54.

Czyzzak, P., & Jaszkiewicz, A. (1998). Pareto simulated annealing – a metaheuristic technique for multiple-objective combinatorial optimization. *Journal of Multi-Criteria Decision Analysis, 7*(1), 34–47.

Dang, G., North, C., & Shneiderman, B. (2001). Dynamic queries and brushing on choropleth maps. In P. Bauersfeld, J. Bennett, & G. Lynch (Eds.), *Proceedings of international conference on information visualization 2001* (pp. 757–764). IEEE Press.

Deb, K. (1999). Multi-objective genetic algorithms: problem difficulties and construction of test problems. *Evolutionary Computation, 7*(3), 205–230.

Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*. Chichester: John Wiley & Sons.

Densham, P. J. (1991). Spatial decision support systems. In D. J. Maguire, M. F. Goodchild, & D. W. Rhind (Eds.). *Geographical information systems: Principle and applications* (Vol. 1, pp. 403–412). Harlow: Longman Scientific and Technical.

Densham, P. J. (1994). Integrating GIS and spatial modelling: visual interactive modelling and location selection. *Geographical Systems, 1*, 203–219.

Dewey, J. (1910). *How we think*. New York: D.C. Heath & Company.

Dibble, C. & Densham, P. J. (1993). Generating interesting alternatives in GIS and SDSS using genetic algorithms. In *GIS/LIS '93*, Minneapolis, Minnesota (pp. 180–189). ACSM, ASPRS, AM/FM International, AAG, URISA.

Duh, J. -D. & Brown, D. G. (this volume). Knowledge-informed Pareto simulated annealing for multiobjective spatial allocation. *Computers, Environment and Urban Systems*, doi:10.1016/j.compenvurbsys.2006.08.002.

Dykes, J., MacEachren, A. M., & Kraak, M.-J. (Eds.). (2005). *Exploring geovisualization*. Kindlington, UK: Elsevier.

Ehrgott, M., & Gandibleux, X. (2000). A survey and annotated bibliography of multiobjective combinatorial optimization. *OR Spektrum, 22*, 425–460.

Eiben, A. E., & Schippers, C. A. (1998). On evolutionary exploration and exploitation. *Fundamenta Informaticae, 35*(1–4), 35–50.

Estivill-Castro, V., & Murray, A. (2000a). Clustering and capacitated facility location via hybrid optimization. In *Intelligent systems and applications*. Canada: ICSC Academic Press.

Estivill-Castro, V., & Murray, A. (2000b). Hybrid optimization for clustering in data mining. In *CLAIO 2000*. Mexico: IMSIO.

Estivill-Castro, V., & Torres-Velázquez, R. (1999). Hybrid genetic algorithm for solving the p-median problem. In X. Yao (Ed.), *Second Asia-Pacific conference on simulated evolution and learning, SEAL'98* (pp. 18–25). Berlin: Springer-Verlag.

Feng, C.-M., & Lin, J.-J. (1999). Using a genetic algorithm to generate alternative sketch maps for urban planning. *Computers, Environment and Urban Systems, 23*, 91–108.

Fogel, L. J. (1962). Autonomous automata. *Industrial Research, 4*, 14–19.

Fonseca, C. M., & Fleming, P. J. (1993). Genetic algorithms for multiobjective optimization: formulation, discussion and generalization. In S. Forrest (Ed.), *Proceedings of the fifth international conference on genetic algorithms* (pp. 416–423). San Mateo, CA: Morgan Kaufmann.

Gahegan, M., Takatsuka, M., Wheeler, M., & Hardisty, F. (2002). Introducing GeoVISTA Studio: an integrated suite of visualization and computational methods for exploration and knowledge construction in geography. *Computers, Environmental and Urban Systems, 26*, 267–292.

Gandibleux, X., Mezdaoui, N., & Fréville, A. (1997). A tabu search procedure to solve multiobjective combinatorial optimization problems. In R. Caballero, F. Ruiz, & R. Steuer (Eds.), *Advances in multiple objective and goal programming. Lecture notes in economics and mathematical systems* (Vol. 455, pp. 291–300). Berlin: Springer.

Garey, M. R., & Johnson, D. S. (1979). *Computers and intractability: a guide to the theory of NP-completeness*. San Francisco: Freeman.

Glover, F., & Laguna, M. (1997). *Tabu search*. Boston: Kluwer Academic Publishers.

Goldberg, D. E. (1989). *Genetic algorithms in search, optimization and machine learning*. Reading, MA: Addison-Wesley.

Grefenstette, J. J., Gopal, R., Rosmaita, B., & van Gucht, D. (1985). Genetic algorithms for the traveling salesman problem. In J. J. Grefenstette (Ed.), *Proceedings of the first international conference on genetic algorithms and their applications* (pp. 160–166). Hillsdale, NJ: Lawrence Erlbaum.

Guimarães Pereira, Â. (1996). Generating alternative routes by multicriteria evaluation and a genetic algorithm. *Environment and Planning B, 23*, 711–720.

Holland, J. H. (1975). *Adaptations in natural and artificial systems*. Ann Arbor, MI: University of Michigan Press.

Horn, J., Nafpliotis, N., & Goldberg, D. E. (1994). A niched Pareto genetic algorithm for multiobjective optimization. *Proceedings of the first IEEE conference on evolutionary computation, IEEE world congress on computational intelligence* (Vol. 1, pp. 82–87). Piscataway, NJ: IEEE Service Center.

Hosage, C. M., & Goodchild, M. F. (1986). Discrete space location-allocation solutions from genetic algorithms. *Annals of Operations Research, 6*, 35–46.

Inselberg, A. (1981). *n*-dimension graphs: Part I – lines and hyperplanes. Technical Report G320-2711, IBM Scientific Center, Los Angeles, CA.

Jankowski, P., Andrienko, N., & Andrienko, G. (2001). Map-centered exploratory approach to multiple criteria spatial decision making. *International Journal of Geographical Information Science, 15*(2), 101–127.

Jankowski, P., & Nyerges, T. (2001). GIS-supported collaborative decision making: results of an experiment. *Annals of the Association of American Geographers, 91*(1), 48–70.

Jaramillo, J. H., Bhadury, J., & Batta, R. (2002). On the use of genetic algorithms to solve location problems. *Computers & Operations Research, 29*, 761–779.

Jones, C. V. (1996). *Visualization and optimization*. Boston: Kluwer Academic Publishers.

Josephson, J. R., & Josephson, S. G. (1994). *Abductive inference: computation, philosophy, technology*. Cambridge: Cambridge University Press.

Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. Jr., (1983). Optimization by simulated annealing. *Science, 220*, 671–680.

Koza, J. R. (1992). *Genetic programming: on the programming of computers by means of natural selection*. Cambridge, MA: The MIT Press.

Krzanowski, R. M., & Raper, J. (1999). Hybrid genetic algorithm for transmitter location in wireless networks. *Computers, Environment and Urban Systems, 23*, 359–382.

Langdon, W. B., & Poli, R. (2002). *Foundations of genetic programming*. Berlin: Springer.

Lin, Y.-C., Hwang, K.-S., & Wang, F.-S. (2001). Co-evolutionary hybrid differential evolution for mixed-integer optimization problems. *Engineering Optimization, 33*, 663–682.

Lotov, A. V., Bushenkov, V. A., & Kameney, G. K. (2004). *Interactive decision maps: approximation and visualization of Pareto frontier*. Boston: Kluwer Academic Publishers.

MacDougall, E. B. (1992). Exploratory analysis, dynamic statistical visualization, and geographical information systems. *Cartography and Geographic Information Systems, 19*(4), 237–246.

MacEachren, A. M., & Kraak, M.-J. (1997). Exploratory cartographic visualization: advancing the agenda. *Computers & Geosciences, 23*(4), 335–343.

Mackinlay, J., Robertson, G., & Card, S. (1991). The perspective wall: Detail and context smoothly integrated. In *Proceedings of CHI'91, ACM conference on human factors in computing systems* (pp. 173–180). ACM Press.

Malczewski, J. (1999). Visualization in multicriteria spatial decision support systems. *Geomatica, 53*, 139–147.

Malczewski, J., Pazner, M., & Zaliwska, M. (1997). Visualization of multicriteria location analysis using raster GIS: a case study. *Cartography and Geography Information Systems, 24*(2), 80–90.

Martin, W. N., Lienig, J., & Cohoon, J. P. (1997). Island (migration) models: evolutionary algorithms based on punctuated equilibria. In T. Bäck, D. B. Fogel, & Z. Michalewicz (Eds.), *Handbook of evolutionary computation* (pp. C6.3:1–C6.3:16). New York: Oxford University Press/IOP.

Melachrinoudis, E., Min, M., & Wu, X. (1995). A multiobjective model for the dynamic location of landfills. *Location Science, 3*(3), 143–166.

Michalewicz, Z. (1996). *Genetic algorithms + data structures = evolution programs*. Berlin: Springer-Verlag.

Miettinen, K. M. (1999). *Nonlinear multiobjective optimization*. Boston: Kluwer Academic Publishers.

Monmonier, M. (1989). Geographical brushing: enhancing exploratory analysis of the scatterplot matrix. *Geographical Analysis, 21*, 81–84.

Nix, A. E., & Vose, M. D. (1992). Modeling genetic algorithms with Markov chains. *Annals of Mathematics and Artificial Intelligence, 5*, 79–88.

Packard, K. (2000). Translucent windows in X. Talk given at Atlanta Linux Showcase, October 2000. In *Proceedings of the 4th Annual Linux Showcase* & *Conference*, Atlanta, GA.

Pareto, V. (1896). *Cours D'Economie Politique*. F. Rouge: Lausanne. English translation published in 1971 by Augustus M. Kelley, New York.

Preux, P., & Talbi, E.-G. (1999). Towards hybrid evolutionary algorithms. *International Transactions in Operational Research, 6*(6), 557–570.

Rechenberg, I. (1965). Cybernetic solution path of an experimental, Problem library translation No. 1122. Royal Aircraft Establishment.

Reeves, C. R. (Ed.). (1993). *Modern heuristic techniques for combinatorial problems*. Oxford: Blackwell Scientific Publications.

Ruiz-Andino, A., Araujo, L., Sáenz, F., & Ruz, J. (2000). A hybrid evolutionary approach for solving constrained optimization problems over finite domains. *IEEE Transactions on Evolutionary Computation, 4*(4), 353–372.

Schilling, D. A., ReVelle, C., & Cohon, J. (1983). An approach to the display and analysis of multiobjective problems. *Socio-Economic Planning Sciences, 17*(2), 57–63, method of value paths.

Simões, A. & Costa, E. (2000). Using genetic algorithms with sexual or asexual transposition: a comparative study. In *Proceedings of the 2000 congress on evolutionary computation (CEC '2000)*, pp. 1196–1203.

Simon, H. A. (1960). *The new science of management decision*. New York: Happer & Row.

Srinivas, N., & Deb, K. (1995). Multiobjective optimization using nondominated sorting in genetic algorithms. *Evolutionary Computation, 2*(3), 221–248.

Stewart, T. J., Janssen, R., & van Herwijnen, M. (2004). A genetic algorithm approach to multiobjective land use planning. *Computers & Operations Research, 31*, 2293–2313.

Swayne, D. F., Cook, D., & Buja, A. (1998). XGobi: interactive dynamic data visualization in the X window system. *Journal of Computational and Graphical Statistics, 7*(1), 113–130.

Tufte, E. R. (1990). *Envisioning information*. Cheshire, CT: Graphics Press.

Tukey, J. W. (1977). *Exploratory data analysis*. Reading, MA: Addison-Wesley.

van Veldhuizen, D. A. (1999). *Multiobjective evolutionary algorithms: classification, analysis, and new innovations*. Wright-Patterson, OH: Air Force Institute of Technology.

Voss, S., Martello, S., Osman, I. H., & Roucairol, C. (1999). *Metaheuristics: advances and trends in local search paradigms for optimization*. Boston: Kluwer.

Williams, J. C. Jr., (1995). Political redistricting: a review. *Papers in Regional Science, 74*(1), 13–40.

Xiao, N. (2006). An evolutionary algorithm for site search problems. *Geographical Analysis, 38*(3), 227–247.

Xiao, N., & Armstrong, M. P. (2003). A specialized island model and its application in multiobjective optimization. In E. Cantú-Paz, J. A. Foster, K. Deb, L. D. Davis, R. Roy, U.-M. O'Reilly, H.-G. Beyer, R. Standish, G. Kendall, S. Wilson, M. Harman, J. Wegener, D. Dasgupta, M. A. Potter, A. C. Schultz, K. A. Dowsland, N. Jonoska, & J. Miller (Eds.), *Genetic and evolutionary computation — GECCO 2003*. *Lecture notes in computer science* (2724, pp. 1530–1540). Berlin: Springer-Verlag.

Xiao, N., & Armstrong, M. P. (2005). Supporting the comparison of choropleth maps using an evolutionary algorithm. *Cartography and Geographical Information Science, 32*(4), 347–358.

Xiao, N., & Armstrong, M. P. (2006). ChoroWare: a software toolkit for choropleth map classification. *Geographical Analysis, 38*(1), 102–121.

Xiao, N., Bennett, D. A., & Armstrong, M. P. (2002). Using evolutionary algorithms to generate alternatives for multiobjective site search problems. *Environment and Planning A, 34*(4), 639–656.

Zhang, X. & Armstrong, M. P. (2005). Using a genetic algorithm to generate alternatives for multi-objective corridor location problems. In *Proceedings of GeoComputation 2005*, Ann Arbor, Michigan.

Zionts, S., & Wallenius, J. (1976). An interactive programming method for solving the multiple criteria problem. *Management Science, 22*(6), 652–663.

Zitzler, E., & Thiele, L. (1999). Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. *IEEE Transactions on Evolutionary Computation, 3*(4), 257–271.