THE WELFARE EFFECTS OF PAY-AS-YOU-GO RETIREMENT PROGRAMS: THE ROLE OF TAX AND BENEFIT TIMING

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It is well known that pay-as-you-go retirement programs reduce steady-state welfare and the capital stock in dynamically efficient overlapping generation (OLG) economies. The common two-period OLG model obscures, however, the relationship between the magnitude of these effects and the ages at which taxes are paid and benefits received. Program changes that shift taxes to older workers or benefits to younger retirees have effects similar to reductions in program size, yielding steady-state welfare gains and increases in capital accumulation while imposing transition costs on current generations. This analysis has policy implications for both tax and benefit timing. (*JEL* H55, E62)

I. INTRODUCTION

The study of two-period overlapping generation (OLG) models has yielded important insights into the welfare effects of pay-as-you-go retirement programs in dynamically efficient economies. A pay-as-you-go program offers windfall gains during its start-up phase but lowers steady-state utility because its steady-state rate of return equals the economy's growth rate, which, under dynamic efficiency, is lower than the marginal product of capital. Shutting down or scaling back the program allows future generations to earn higher returns but imposes a transition cost on current generations who have paid into the program but have not yet received full benefits. The future generations' gains and the transition cost are equal in present value. It is well known that these results extend to continuous-time and multiperiod OLG models.

I show, however, that the two-period model fails to capture the role of one important factor. The magnitude of a pay-as-you-go program's welfare effects depends on its life cycle timing—the ages at which each cohort pays taxes and receives benefits. In the two-period model, taxes must be paid in "Period 1" and benefits must be received in "Period 2." In contrast, actual programs have flexibility in the allocation of taxes within the working lifetime and benefits within the retirement years. I show that the program's steady-state welfare loss is smaller when taxes are paid at later ages or benefits received at earlier ages.

These effects arise because the pay-as-you-go program's rate-of-return shortfall is less harmful to each cohort when compounded over a shorter or later time period. Shifting taxes to older ages or benefits to younger ages therefore aids future generations in a manner similar to reducing the size of the pay-as-you-go program. Like a reduction in program size, though, such a timing shift imposes a transition cost on current generations, equal in present value to future generations' gains. Specifically, shifting taxes to older workers boosts lifetime taxes for some of the cohorts working at the time of implementation, while shifting benefits to younger retirees reduces lifetime benefits for some of the cohorts retired at that time.

In a simple calibration of the U.S. Social Security program, instituting a payroll tax exemption during the first 10 yr of working life (with a revenue-neutral tax increase on older

ABBREVIATIONS
CGL: Closed-Group Liability
CGL_t: Period *t* Closed-Group Liability
OLG: Overlapping Generation

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workers) would reduce the program's steady-state welfare loss by about one-sixth but would raise lifetime taxes for most of the cohorts working at the time of implementation. Policy changes that raised benefits for younger retirees (with a budget-neutral benefit reduction for older retirees) would also generate steady-state welfare gains and transition costs, but of smaller magnitudes.

In Section II, I review the familiar analysis of pay-as-you-go retirement programs in two-period OLG models. In Section III, I explain the role of tax and benefit timing in a continuous-time OLG model under the assumption that all taxes are paid at one age and all benefits are received at another age. In Section IV, I show that these results generalize to the more realistic case in which taxes and benefits are paid at multiple ages. I examine the implications for tax timing in Section V and those for benefit timing in Section VI. Section VII concludes.

II. REVIEW OF TWO-PERIOD MODELS

I begin by reviewing the basic properties of pay-as-you-go retirement programs in the familiar two-period OLG model. Technology is linear, implying fixed factor prices, and labor supply is inelastic. In each period t, the number of workers is N^t and the per-worker wage G^t. The gross-of-principal one-period marginal product of capital is R. I assume $R > NG$, so that the economy is dynamically efficient.

Consider a simple pay-as-you-go retirement program. In period t, each of the N^t workers pays τG^t as a lump-sum tax and each of the N^{t-1} retirees receives τNG^t as a lump-sum benefit.[1] The lifetime present value net burden on each period t worker is

$$(1) \qquad \tau G^t\left\{1-\frac{NG}{R}\right\}.$$

The money's worth ratio, the present value of benefits divided by that of taxes, equals (NG/R).

Since the present value in Equation (1) would equal 0 if R equaled NG, the program's internal rate of return is NG, the economy's growth rate.[2] With $R > NG$, the continued operation of the pay-as-you-go program places a burden on each future generation.[3] The period t closed-group liability, CGL_t, is the period t present value of the aggregate burden that continuation imposes on period t workers and later generations (equivalently, their gain from ending the program). Multiplying $\tau G^s\{1-(NG/R)\}$, the burden on each period s worker, by cohort size N^s and discount factor R^{t-s} and summing across s from t to infinity yield

$$(2) \qquad \mathrm{CGL}_t = \tau(NG)^t.$$

Although abruptly ending the program at the beginning of period t benefits period t workers and later generations, it imposes a "transition cost" on the period t retirees. This generation, which has already paid its taxes, loses its benefits, which have an aggregate value of $\tau(NG)^t$. Note that this transition cost is equal to the present value of future generations' gains, which is the closed-group liability (CGL) given by Equation (2). This present-value equality also applies to the initial creation of the program—the present value of the program's burden on future generations equals the start-up gains of the initial retirees, who receive benefits without paying taxes. It can be shown that the equality also applies to gradual and phased-in changes.[4]

As discussed by Lindbeck and Persson (2003, 82), Kotlikoff (2002, 1878–1886), and Auerbach and Kotlikoff (1987, 148), the pay-as-you-go program also depresses capital accumulation. With endogenous factor prices, the reduction in the capital stock results in lower wages and a higher marginal product of capital.

It is well known that the two-period model's basic insights extend to continuous-time

1. As befits a pure pay-as-you-go program, budget balance is assumed to hold in each period. This assumption rules out the small temporary surpluses and deficits often posted by programs that are essentially pay-as-you-go, such as the post-1983 U.S. Social Security surpluses.

2. This result was derived by Samuelson (1958) and Aaron (1966). In addition, see Lindbeck and Persson (2003, 79), Feldstein and Liebman (2002, 2257–2258), Geanakoplos, Mitchell, and Zeldes (1999, 84), and Auerbach and Kotlikoff (1987, 147–148).

3. In contrast, if $R <$ NG (the economy is dynamically inefficient), the pay-as-you-go retirement program increases all generations' well-being. Abel, Mankiw, Summers, and Zeldes (1989) provide evidence, though, that the U.S. economy is dynamically efficient.

4. For further discussion, see Lindbeck and Persson (2003, 80–81), Kotlikoff (2002, 1881–1882), Feldstein and Liebman (2002, 2258–2259), and Geanakoplos, Mitchell, and Zeldes (1999, 86–87).

models: the program's steady-state return equals the economy's growth rate, the program lowers steady-state welfare (under dynamic efficiency), and ending the program imposes a transition cost on current generations equal to the CGL. I show, however, that continuous-time models also provide an important role for life cycle timing effects that are suppressed in the two-period model.

III. TWO-AGE PROGRAMS IN CONTINUOUS TIME

I examine a continuous-time overlapping generations economy with linear technology. At date t, $\exp(nt)$ people begin economic life and live for a fixed period L. The age a population is $\exp[n(t-a)]$ and the total population $P\exp(nt)$, where $P \equiv [1-\exp(-nL)]/n$. At date t, the per-capita wage equals $\exp(gt)$ and national labor income equals $P\exp[(n+g)t]$. The marginal product of capital is r. I assume $r > n + g$, so that the economy is dynamically efficient.

Consider a simple "two-age" pay-as-you-go retirement program that collects taxes solely at age A_T and pays benefits solely at age $A_B > A_T$. Transfers are equal to a fixed fraction τ of national labor income. At date t, each of the $\exp[n(t-A_T)]$ individuals aged A_T pays tax of $\tau P\exp(gt + nA_T)$ and each of the $\exp[n(t-A_B)]$ individuals aged A_B receives benefit of $\tau P\exp(gt + nA_B)$. Each individual entering the economy at date s then faces a net lifetime burden, with a date s present value of (PVT − PVB) $\exp(gs)$, where

$$\text{(3)} \qquad \begin{aligned} \mathrm{PVT} &\equiv \tau P\exp[(n+g-r)A_T], \\ \mathrm{PVB} &\equiv \tau P\exp[(n+g-r)A_B]. \end{aligned}$$

A. Steady-State Effects of Tax and Benefit Timing

As in the two-period model, the steady-state burden is proportional to the program's size τ and is increasing in the rate-of-return shortfall, $r - n - g$. But the burden is also greater if the tax age A_T is lower or the benefit age A_B is higher. These tax and benefit timing effects, which are the focus of this article, are not captured by the two-period model, in which tax payment and benefit receipt must occur in Period 1 and Period 2, respectively.

Specifically, the money's worth ratio, the present value ratio of benefits to taxes, is $\exp[(n+g-r)(A_B - A_T)]$, which has a straightforward interpretation. It is the ratio of the present value of the payout to the initial investment outlay for an individual required to invest in an asset paying $n + g$ rather than the market return r for an interval of length $A_B - A_T$. Individuals experience similar effects when they participate in a pay-as-you-go program, with return $n + g$. In either context, a rate-of-return shortfall is more harmful when compounded over a longer interval.

To be more concrete, set $n + g$ equal to .03 (reflecting 1% population growth and 2% productivity growth) and r equal to .05 (a conservative estimate of the marginal product of capital). Investing at 3% rather than 5% over a 1-yr interval is only slightly harmful; the money's worth ratio equals $\exp(-0.02)$ or .98, so only 2% of the investment is lost due to the below-market return. Investing at such returns over a 10-yr interval is considerably more harmful; the money's worth ratio equals $\exp(-0.2)$ or .82 and the loss is 18%. Over a 30-yr period, the harm is much greater, with a money's worth ratio of $\exp(-0.6)$ or .55, yielding a 45% loss.

A pay-as-you-go program imposes similarly small steady-state welfare losses if there is only a 1-yr gap between taxes and benefits; if, say, social security taxes were paid at age 50 and benefits received at age 51, there would be little loss from the below-market returns. The losses are much greater if taxes are paid at age 40 and benefits received at age 70.

Expression (3) also reveals that, for any given values of the interval $A_B - A_T$ and the program size τ, the absolute burden is smaller if the tax and benefit ages are later. Delaying both the tax and the benefit ages by 1 yr reduces each present value by 2%, which leaves the money's worth ratio unchanged but reduces the size of the net burden by 2%. The beginning-of-life present value of the burden is therefore 2% smaller if taxes are paid at age 41 and benefits received at age 51 than if taxes are paid at age 40 and benefits received at age 50.

This result can be understood by again considering the investment analogy. The beginning-of-life present value burden of a required below-market investment depends on the beginning-of-life present value of the amount invested. In this case, delaying each individual's tax by 1 yr while holding τ fixed reduces the beginning-of-life present value of the tax by 2%. The 1-yr delay raises the size of the tax payment by 3% (since revenue remains a fixed fraction τ of national labor income, which grows 3% each year) and the present value of the tax then falls by 2% because it is discounted an additional 5%.

B. Present-Value Equality Continues to Hold

As is well known, the present-value equality also holds in continuous-time models. In this two-age case, abruptly shutting down the program imposes a transition cost on individuals aged between A_{T} and A_{B}, who are denied their benefit despite having paid their tax. Of course, individuals younger than A_{T} gain (in the same manner as future cohorts); their gains must be subtracted to obtain the net transition cost on current cohorts. As shown in Section A of the Appendix, both the CGL (the welfare gain to future cohorts) and the net transition cost are equal to $(\mathrm{PVT}-\mathrm{PVB})\exp[(n+g)t]/(r-n-g)$ for a date t shutdown. It can be shown that, as before, the equality also holds for gradual and phased-in changes.

As mentioned above, the two-age program has a smaller steady-state welfare loss when A_{T} and A_{B} are closer to each other or when both ages are higher. The present-value equality therefore dictates that the transition cost from shutting down the program must then also be smaller. This result can easily be confirmed. When A_{T} and A_{B} are closer to each other, fewer cohorts are aged between A_{T} and A_{B} and therefore lose from the shutdown. When both ages are higher, more of the current cohorts are younger than A_{T} and therefore gain from the shutdown, reducing the net transition cost for current cohorts as a group.

As in the two-period model, the present-value equality also applies to program start-up; the present value of the program's total burden on future cohorts equals the start-up gains of the initial cohorts who receive benefits without paying taxes. The burden imposed on future cohorts by a program that collects taxes at age 40 and pays benefits at age 41 is small; the start-up bonus offered by its abrupt introduction is also small because only those aged between 40 and 41 yr at that time receive benefits without paying taxes. If the two ages were further apart, the steady-state welfare loss would be larger, as would the start-up bonus since a larger number of cohorts would receive benefits without paying taxes. In addition, changing the tax and benefit ages from 40 and 41 yr to 50 and 51 yr would yield a reduction in the start-up bonus, along with a reduction in the steady-state welfare loss. With these higher ages, the program's introduction would harm a larger number of initial cohorts (all cohorts younger than 50 yr, rather than only those younger than 40 yr), thereby reducing the net start-up bonus received by the initial cohorts as a group.

The present-value equality has another important implication. Because the equality holds for any pair of tax and benefit ages, it also holds for any change from one pair to another. By raising the tax age or lowering the benefit age, policymakers can reduce the program's steady-state burden while avoiding a reduction in its size, but they cannot avoid the transition cost. Abruptly raising the tax age from 40 to 50 yr imposes a transition cost on workers then aged between 40 and 50 yr; they are taxed again at age 50 under the new rules, after having been taxed at age 40 under the old rules. Abruptly lowering the benefit age from, say, 70 to 60 yr imposes a transition cost on retirees then aged between 60 and 70 yr; they were too young to receive benefits under the old rules but are too old to receive benefits under the new rules. As before, the transition cost cannot be eliminated through gradual and phased-in changes. As in the two-period model, there is no free lunch.

IV. GENERAL CASE

The above analysis assumes that taxes are paid at a single age and benefits received at another single age. I show that the conclusions extend to programs that collect taxes and pay benefits at a variety of ages.[5]

Assume that at each date t, each of the $\exp[n(t-a)]$ individuals aged a pays $T(a)$ $\exp[g(t-a)]$ or receives $B(a)\exp[g(t-a)]$. The budget constraint $\int_0^L T(a)\exp[-(n+g)a]\mathrm{d}a=\int_0^L B(a)\exp[-(n+g)a]\mathrm{d}a=\tau P$ ensures that aggregate taxes and benefits each equal fraction τ of national labor income.[6] Then, the present value burden for an individual

5. Section B of the Appendix confirms that the present-value equality holds in this general case.

6. In the two-age case, $T(a)$ is $\tau P\exp[(n+g)A_T]$ at A_T and zero elsewhere, while $B(a)$ is $\tau P\exp[(n+g)A_B]$ at A_B and zero elsewhere.

entering the economy at date t is (PVT − PVB) $\exp(gt)$, where

$$\text{(4)} \qquad \text{PVT} \equiv \int_0^L T(a)\exp(-ra)\mathrm{d}a,$$
$$\text{PVB} \equiv \int_0^L B(a)\exp(-ra)\mathrm{d}a.$$

Taking a first-order Taylor approximation to the logs of PVT and PVB with respect to r, evaluated at r equal to $n + g$, and using the budget constraint yield

$$\text{(5)} \qquad \ln \text{PVT} \approx \ln(\tau P) - (r - n - g)A_{\text{T}},$$
$$\ln \text{PVB} \approx \ln(\tau P) - (r - n - g)A_{\text{B}},$$

where $A_{\text{T}} \equiv \int_0^L aT(a)\exp[-(n+g)a]\mathrm{d}a / \int_0^L T(a)\exp[-(n+g)a]\mathrm{d}a$, $A_{\text{B}} \equiv \int_0^L aB(a)\exp[-(n+g)a]\mathrm{d}a / \int_0^L B(a)\exp[-(n+g)a]\mathrm{d}a$. (From the budget constraint, the denominators of the A_{T} and A_{B} expressions both equal τP.)

Taking the exponential of Equation (5) yields an expression identical to Equation (3), the corresponding two-age expression. Up to a Taylor approximation error, the analysis is unchanged, except that A_{T} and A_{B} are now weighted average ages of tax payment and benefit receipt, respectively, rather than the single ages previously considered.

In the weighted averages that define A_{T} and A_{B}, each age is weighted by the present value of taxes or benefits at that age, using the discount rate $n + g$ (the value around which the Taylor approximation is taken). These weighted averages are algebraically identical to the bond duration measure of Macaulay (1938, 48–50), which is prominent in the bond pricing literature. Macaulay duration is a weighted average of the time remaining until a bond's future payments, with weights given by the present value of each payment. The economic interpretation is the same in both contexts; just as a bond's duration governs the sensitivity of its present value (price) to the interest rate, so do these weighted average ages govern the sensitivity of the present values PVT and PVB to the discount rate r.

To obtain more specific results and to avoid reliance on the Taylor approximation, I calibrate a stylized representation of the U.S. Social Security old age and survivor (but not disability) program, as further detailed in Section C of the Appendix. I continue to set n to .01, g to .02, and r to .05. I assume that individuals work from economic ages 0 to X and are retired from economic ages X to L. I set X equal to 42 and L equal to 60, corresponding to work from the biological ages 20 to 62 and retirement from the biological ages 62 to 80. The population size parameter P is then 45.1.

Under the benchmark policy, the program is financed by an age-uniform payroll tax of rate τ, implying that the timing of tax payments matches that of wages. I fit a quadratic, cross-sectional, age-earnings profile to recent data. In addition, under the benchmark policy, benefits are paid from ages X to L and remain unchanged in real terms for each cohort throughout retirement.

In this benchmark case, PVT equals 29.89τ and PVB equals 16.62τ. These values are the same as those for a two-age program with A_{T} equal to 20.6 (roughly the midpoint of working life) and A_{B} equal to 49.9 (close to the midpoint of the retirement period). The money's worth ratio is .556. The CGL is 14.7 times the annual benefit payments, which would correspond to a value of \$6.4 trillion in 2005.

Starting from the benchmark policy, I first examine reforms that alter the timing of taxes and then I consider reforms that change the timing of benefits.

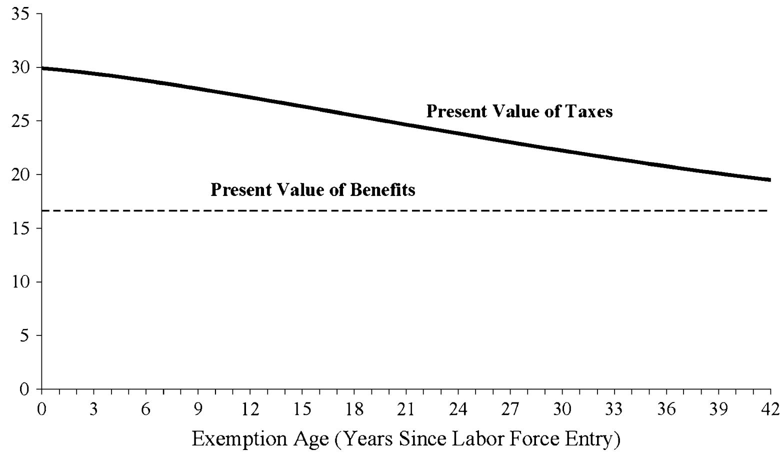
V. TAX TIMING CHANGES

Consider a revenue-neutral replacement of the age-uniform payroll tax with a young-worker exemption policy. As detailed in Section D of the Appendix, such a policy imposes no tax on the earnings of workers below a specified economic age Y. To maintain revenue neutrality, it taxes earnings from ages Y through X at a rate higher than τ. Figure 1 shows the steady-state tax burden PVT for exemption ages from 0 (the benchmark policy) to 42 (the extreme policy in which each worker pays his or her lifetime taxes in a large single payment right before retirement). As the exemption age rises, the lifetime present value of the tax burden falls, in accordance with the analysis presented above.

Consider Y equal to 10, so that workers are exempted from taxes during the first 10 yr of working life (biological ages 20–30 yr), with a revenue-neutral increase in the tax rate on older workers to 1.21τ. Then, PVT is reduced

FIGURE 1

Present Value of Taxes With Young-Worker Exemption (Multiple of Annual Per-Capita Tax At Date of Labor Force Entry)



from 29.89τ to 27.72τ, which is equivalent to raising A_T from 20.6 to 24.4 in a two-age program. Since PVB still equals 16.62τ, the net steady-state lifetime loss from the pay-as-you-go system falls from 13.27τ to 11.10τ, a reduction of more than 16%. In other words, the steady-state gain from this young-worker exemption is equal to the gain that would be attained by scaling back the system by one-sixth across the board. This gain corresponds to a reduction in the CGL of about $1.0 trillion in 2005.

In accordance with the present-value equality, however, the gain to future generations is accompanied by a transition cost of the same size on current generations. If exempting workers from taxes during their first 10 working years yields the same steady-state gains as shrinking the program by one-sixth, then it must impose the same aggregate transition costs on current generations. The two policies allocate the transition cost differently; retirees would bear much of the cost under the program shrinkage, while current workers bear the full cost under the young-worker exemption.

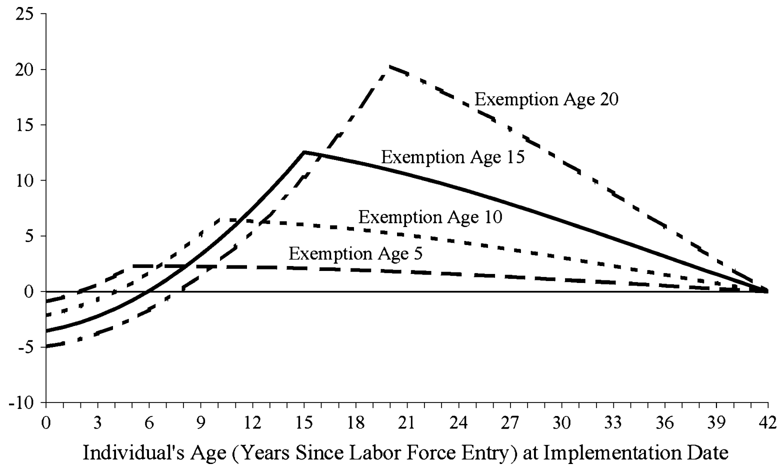
Figure 2 plots the present value loss (negative if gain) borne by each member of the various working cohorts from the abrupt introduction of young-worker exemptions, with exemption ages of 5, 10, 15, and 20 yr. For Y equal to 10, workers aged 0–4.0 yr are net winners and older workers net losers. In general, the loss is greatest for workers around the exemption age since workers at that age obtain no gain from the exemption and have the longest exposure to the higher tax rate.

By assuming particular utility and production functions (as detailed in Section E of the Appendix), it is possible to compute the general equilibrium effects on capital accumulation in an economy with endogenous factor prices. For this purpose, I set τ equal to .056, the 2005 ratio of old age and survivor benefits to national labor income. Figure 3 plots the increases in the steady-state capital stock resulting from young-worker exemptions for ages 0–42 yr. For comparison, the figure shows the 8.0% increase that would arise from shutting down the pay-as-you-go program.

The relative effects of the various policies are virtually unchanged from those computed above in the partial equilibrium fixed factor prices framework. For example, a young-worker exemption with Y equal to 10 increases the steady-state capital stock by 1.2%, which is 15% of the increase attained from shutting down the program; recall that, in the fixed factor price computations, this policy yielded 16% of the steady-state welfare gains offered by a program shutdown.

These capital accumulation effects follow straightforwardly from the analysis of Seidman and Lewis (2003), who showed that any revenue-neutral shift of the tax burden

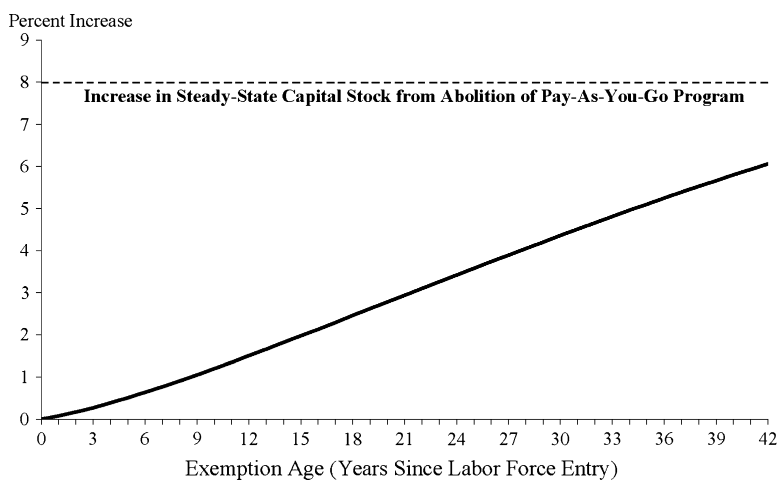
FIGURE 2
Individuals' Net Losses From Young-Worker Exemptions (Multiple of Annual Per-Capita Tax at Implementation Date)



from young to old increases the steady-state capital stock. As they noted, a similar point had been made by authors studying the choice between consumption and wage taxation, including Auerbach and Kotlikoff (1987, 58–60) and Summers (1981). The present analysis applies this general insight to payroll taxation, thereby linking the analysis to the literature on pay-as-you-go retirement programs and permitting an extension to benefit timing. This analysis, like much of the pay-as-you-go literature, emphasizes the partial equilibrium welfare effects, with less attention to the capital accumulation effects emphasized by Seidman and Lewis. The difference in emphasis is largely a matter of taste since the two effects are inextricably linked.

Hubbard and Judd (1987) presented a separate argument for a young worker payroll tax exemption based on borrowing restrictions.

FIGURE 3
Increase in Steady-State Capital Stock From Young-Worker Exemption



(For a related analysis, see Hurst and Willen [2004].). Hubbard and Judd assumed a pre-funded social security system that pays the market return r. They noted that if young workers face binding borrowing restrictions, their shadow interest rates exceed r, and it is then desirable to delay tax payments. In contrast, the present analysis assumes no borrowing restrictions, so that the workers' shadow interest rate equals r, but considers a pay-as-you-go system that pays a rate of return $n + g < r$. If these complementary analyses are combined, the steady-state welfare gain from a young-worker exemption is even larger, as borrowing restrictions push workers' shadow rate above r, while the pay-as-you-go system offers a return lower than r.

VI. CHANGES IN BENEFIT TIMING

The analysis mentioned in the preceding section considered the steady-state gains and transition costs associated with delaying tax payments. As discussed earlier, qualitatively similar effects can be achieved by accelerating benefit receipt. Although the effects of benefit timing changes are generally smaller than those of tax timing changes, they can still be significant. I consider three policies to alter benefit timing, which are described in detail in Section F of the Appendix.

The easiest way to compare the effects of tax and benefit timing changes is to consider the (unrealistic) policy that is analytically parallel to the young-worker exemption: an old-retiree cutoff that eliminates benefits for retirees above a specified economic age J, with a budget-neutral benefit increase for younger retirees. Calculations reveal that old-retiree cutoffs have smaller effects than young-worker exemptions. For example, the extreme policy of paying all lifetime benefits at the onset of retirement (J equal to 42) raises PVB only from 16.62τ to 19.48τ, a gain of 2.86τ; the corresponding extreme policy of collecting all lifetime taxes at that same age (Y equal to 42) lowers PVT from 29.89τ to 19.48τ, a gain of 10.41τ, over three times larger.[7] Similarly, denying benefits during the second half of retirement (J equal to 51) raises PVB by 1.28τ, while eliminating taxes during the first half of working life (Y equal to 21) lowers PVT by 5.25τ.

The smaller impact of benefit timing changes is easily explained. Raising A_T by 1 yr and lowering A_B by 1 yr have the same proportional effects, reducing PVT by 2% and increasing PVB by 2%. But since PVT is almost twice as large as PVB under the benchmark policy, the tax change has a larger absolute effect. Moreover, because working life is longer than retirement (here, 42 versus 18 yr), the increases in A_T from the above tax changes are larger than the reductions in A_B from the above benefit changes. For example, the extreme tax timing policy raises A_T by 21.6 yr, from 20.4 to 42 yr, while the extreme benefit timing policy lowers A_B by only 7.9 yr, from 49.9 to 42 yr.

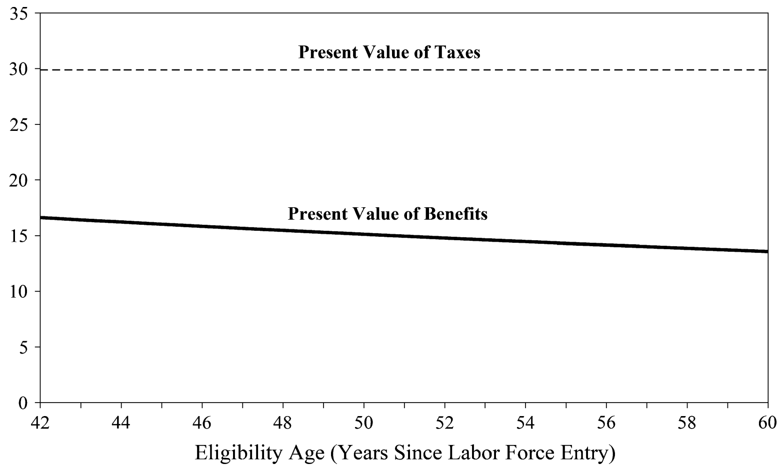
I next consider changes in the benefit growth rate during retirement. Under the benchmark policy (as in the actual social security system), each individual's real benefits remain constant throughout retirement. A budget-neutral change that hikes initial benefits but then lets them fall 4% per year throughout retirement raises PVB from 16.62τ to 16.96τ. Conversely, a budget-neutral change that reduces initial benefits and then lets them rise 4% per year lowers PVB to 16.27τ.

Finally, I consider a policy parameter that has been changed in past social security reforms, the benefit eligibility age. Relative to leaving the program unchanged, an eligibility-age increase obviously shrinks the program and reduces the steady-state welfare loss. Because the eligibility-age increase delays benefit receipt, however, it reduces the steady-state burden by less than a budget-equivalent across-the-board benefit reduction that applies to retirees of all ages.

To examine this issue, consider a budget-neutral eligibility-age increase in which benefits are delayed until $V > X$ and are increased for older retirees. As shown in Figure 4, denying benefits during the first 3 yr of retirement (V equal to 45) lowers PVB from 16.62τ to 16.03τ. Although changes in benefit timing have smaller impacts than those in tax timing, the effects can still be significant. These computations suggest that a 3-yr eligibility-age increase would result in a present value aggregate loss for future generations of almost \$300 billion in 2005, compared to across-the-board

7. The combination of the two extreme policies equates PVB to PVT at 19.48τ, eliminating the steady-state welfare loss. Indeed, this combination effectively shuts down the program since there are no real effects from paying taxes that are immediately and fully refunded in the form of benefits.

FIGURE 4
Present Value of Benefits Under Various Eligibility Ages (Multiple of Annual Per-Capita Tax At Date of Labor Force Entry)



cuts that lowered aggregate benefits by the same amount.

This analysis assumes known lifetimes. In a world with uncertain lifetimes and imperfect annuitization, the policies considered in this section, particularly the old-retiree cutoff, would harm individuals who enjoy unexpectedly long lifetimes. Such effects would have to be considered in a more complete analysis.[8] In conjunction with the smaller impact of benefit timing changes, this consideration suggests that tax timing changes are of greater policy relevance.

VII. CONCLUSION

In a continuous-time OLG model, a pay-as-you-go retirement program causes a larger steady-state welfare loss when taxes are paid earlier or benefits received later. The larger loss arises because the pay-as-you-go program's rate-of-return shortfall is more harmful to each cohort when compounded over a longer or earlier time period. Policy changes that exempt younger workers from payroll taxes (with a revenue-neutral tax increase on older workers) increase steady-state welfare but impose transition costs on older workers when implemented. Policy changes that increase benefits for younger retirees (with a budget-neutral benefit cut for older retirees) have similar but smaller effects. Policy analyses of proposed changes in pay-as-you-go retirement programs should consider how the changes affect tax and benefit timing.

8. Feldstein (1990) considers the choice of benefit growth rates in a four-period OLG model, analyzing both the effects considered here and the effects of uncertain lifetimes and imperfect annuitization.

APPENDIX

Section A: CGL and Transition Cost in Two-Age Case

To obtain the date t CGL, apply discount factor $\exp[r(t-s)]$ to the burden (PVT − PVB) $\exp(gs)$ on each of the $\exp(ns)$ date s entrants and integrate across s from t to infinity to obtain $(\text{PVT}-\text{PVB})\exp[(n+g)t]/(r-n-g)$.

The net transition cost is computed as follows. For each age a between A_T and A_B, $\exp[n(t-a)]$ individuals lose $\tau P\exp[g(t-a)+(n+g)A_B]$ at date $t+A_B-a$; the cohort's payments have date t present value $\tau P\exp[(n+g)t+(n+g-r)(A_B-a)]$. Integrating across a from A_T to A_B yields gross transition cost $\tau P\{1-\exp[(n+g-r)(A_B-A_T)]\}\exp[(n+g)t]/(r-n-g)$. For each age a less than A_T, $\exp[n(t-a)]$ individuals avoid a burden with date t present value $\tau P\{\exp[(n+g-r)A_T]-\exp[(n+g-r)A_B]\}\exp[g(t-a)+ra]$. Integrating across a from 0 to A_T yields $\tau P\{\exp[(n+g-r)A_T]-\exp[(n+g-r)A_B]\}\{\exp[(r-n-g)A_T]-1\}\exp[(n+g)t]/(r-n-g)$ or $\tau P\{\exp[(r-n-g)A_B]+1-\exp[(n+g-r)(A_B-A_T)]-\exp[(n+g-r)A_T]\}\exp[(n+g)t]/(r-n-g)$. Subtracting this quantity from the gross transition cost yields $\tau P\{\exp[(n+g-r)A_T]-\exp[(n+g-r)A_B]\}\exp[(n+g)t]/(r-n-g)$, which equals $(\text{PVT}-\text{PVB})\exp[(n+g)t]/(r-n-g)$.

Section B: CGL and Transition Cost in General Case

The date t CGL, in terms of PVT and PVB, is computed in the same manner as in Section A of this Appendix.

The transition cost is computed as follows. The combined date t present value loss of the cohorts aged 0 through L as of date t equals $\exp[(n+g)t]\ \int_0^L \exp[-(n+g)h](\int_x^L \exp[r(h-a)][T(a)-B(a)]\mathrm{d}a)\mathrm{d}h$, which equals $\exp[(n+g)t]\int_0^L \exp(-ra)(\int_0^a \exp[(r-n-g)h]\mathrm{d}h)[T(a)-B(a)]\mathrm{d}a$ or $\exp[(n+g)t]\int_0^L\{\exp[-(n+g)a]-\exp[-ra]\}[T(a)-B(a)]\mathrm{d}a/(r-n-g)$. Since $\int_0^L \exp[-(n+g)a][T(a)-B(a)]\mathrm{d}a=0$ from the budget constraint and $\int_0^L \exp[-ra]\ [T(a)-B(a)]\mathrm{d}a=\mathrm{PVT}-\mathrm{PVB}$, this expression can be rewritten as $(\mathrm{PVB}-\mathrm{PVT})\exp[(n+g)t]/(r-n-g)$.

Section C: Calibration of Social Security System

I assume that each individual of age a (from 0 to X) at date t receives earnings $P(1+w_1a+w_2a^2)\exp(gt)/Z(n+g,0,X)$, where $Z(n+g,0,X)$ is a scaling factor, defined as follows. For any discount rate ρ, any starting age α, and any ending age ω, the scalar $Z(\rho,\alpha,\omega)$ is the age-zero present value (at discount rate ρ) of $(1+w_1a+w_2a^2)\exp(ga)$ between the specified ages,

$$\begin{aligned}Z(\rho,\alpha,\omega)\equiv&\int_\alpha^\omega(1+w_1a+w_2a^2)\exp[(g-\rho)a]\mathrm{d}a\\&=\{\exp[(g-\rho)\alpha]-\exp[(g-\rho)\omega]\}\{(\rho-g)^{-1}\\&\quad+(\rho-g)^{-2}w_1+2(\rho-g)^{-3}w_2\}\\&\quad+\{\alpha\exp[(g-\rho)\alpha]-\omega\exp[(g-\rho)\omega]\}\\&\quad\times\{(\rho-g)^{-1}w_1+2(\rho-g)^{-2}w_2\}\\&\quad+\{\alpha^2\exp[(g-\rho)\alpha]-\omega^2\exp[(g-\rho)\omega]\}\\&\quad\times(\rho-g)^{-1}w_2.\end{aligned}$$

The $Z(n+g,0,X)$ term in the denominator of the earnings function scales wages to keep the per-capita wage equal to $\exp(gt)$. So, $T(a)=\tau P(1+w_1a+w_2a^2)\exp(ga)/Z(n+g,0,X)$ and PVT equals $\tau PZ(r,0,X)/Z(n+g,0,X)$.

Using Social Security Administration data on workers and earnings and Census Bureau data on population, all for 2003, I constructed a proxy for per-capita earnings (number of workers multiplied by median earnings divided by population) for each 5-yr age cohort between ages 20 and 60 and for the 2-yr cohort aged 61–62 yr. I regressed this proxy on a constant, the midpoint economic age of each group (treating biological age 20 as economic age 0), and age squared. I then rescaled the coefficients to set the intercept to 1, obtaining w_1 equal to .2608 and w_2 equal to $-.00511$.

The budget constraint then requires that $B(a)=\tau P(n+g)/\{\exp[-(n+g)X]-\exp[-(n+g)L]\}$ for a from X to L. Then, PVB equals $\tau P(n+g)/(r)\ [\exp(-rX)-\exp(-rL)]/\{\exp[-(n+g)X]-\exp[-(n+g)L]\}$

Section D: Young-Worker Exemption

To maintain revenue neutrality, the young-worker exemption policy must tax ages Y through X at rate $\tau Z(n+g,0,X)Z(n+g,Y,X)$. So, $T(a)$ equals 0 for a less than Y and equals $\tau P(1+w_1a+w_2a^2)\exp(ga)/Z(n+g,Y,X)$ for a from Y through X. Then, PVT equals $\tau P\,Z(r,Y,X)/Z(n+g,Y,X)$.

Assume that the policy is abruptly introduced at date t. Then, for each a between Y and X, each of $\exp[n(t-a)]$ workers suffers a loss with present value $\tau P\{[Z(n+g,0,X)/Z(n+g,Y,X)]-1\}[Z(r,a,X)/Z(n+g,0,X)]\exp[ra+g(t-a)]$ from the higher tax rate. For each a between 0 and Y, each of $\exp[n(t-a)]$ workers of age a suffers a loss with present value $\tau P\{[Z(n+g,0,X)/Z(n+g,Y,X)]-1\}[Z(r,Y,X)/\ Z(n+g,0,X)]\exp[ra+g(t-a)]$ from the higher tax rate they will face after attaining age Y but has a gain with present value $\tau P[Z(r,a,Y)]/[Z(n+g,0,X)]\exp[ra+g(t-a)]$ from the exemption enjoyed until age Y.

Section E: Equilibrium Capital Stock

Letting K denote the capital stock divided by national labor income, the steady-state values of K and r satisfy two conditions. First, $K=\theta/[(1-\theta)(r+\delta)]$, where θ is the capital share in a Cobb-Douglas production function for gross-of-deprecation output and δ is the deprecation rate. Second, K equals the aggregate stock of national saving, which is the present value of future consumption minus that of future disposable noncapital income,

$$K=\int_0^L\exp[-(n+g)h]\left\{\int_h^L\exp[r(h-a)][C(a)-W(a)-B(a)+T(a)]\mathrm{d}a\right\}\mathrm{d}h,$$

where $C(a)$, and $W(a)$ are capital holdings, consumption, and pretax labor income of each worker at age a, respectively. (Like $B(a)$ and $T(a)$, they are expressed as a fraction of the per-capita wage when the individual is of economic age 0.) If consumers maximize $\int_0^L\exp[-\lambda a]\ln(C(a))\mathrm{d}a$, then $C(a)=\{\lambda\exp[(r-\lambda)a]\}/[1-\exp(-\lambda L)]\int_0^L\exp(-rh)[W(x)+B(x)-T(x)]\mathrm{d}h$. Setting θ to .35, δ to .04, and τ equal to .056, I solved backward to find that a time preference rate λ of .018 yields a capital stock consistent with r equal to .05 under the age-uniform benchmark policy. I then computed the equilibrium values of K and r under program abolition and the various young-worker exemptions.

Section F: Changes in Benefit Timing

A budget-neutral old-retiree cutoff with cut-off age J sets $B(a)$ equal to $\tau P(n+g)/\{\exp[-(n+g)X]-\exp[-(n+g)J]\}$ for a from X through J and 0 for a from J through L. Then, PVB equals $\tau P(n+g)/(r)[\exp(-rX)-\exp(-rJ)]/\{\exp[-(n+g)X]-\exp[-(n+g)J]\}$

A budget-neutral change that lets each cohort's real benefit grow at rate q sets $B(a)$ equal to $\tau P(n+g-q)/\{\exp[(q-n-g)X]-\exp[(q-n-g)L]\}\ \exp(qa)$ for a from X to L. Then, PVB equals $\tau P(n+g-q)/(r-q)\{\exp[(q-r)X]-\exp[(q-r)L]\}/\{\exp[(q-n-g)X]-\exp[(q-n-g)L]\}$

A budget-neutral increase in the eligibility age to V sets $B(a)$ equal to 0 for a from X through V and equal to $\tau P(n+g)/\{\exp[-(n+g)V]-\exp[-(n+g)L]\}$ for a from V through L. Then, PVB equals $\tau P[(n+g)/(r)][\exp(-rV)-\exp(-rL)]/\{\exp[-(n+g)V]-\exp[-(n+g)L]\}$

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