



# Modeling gentrification dynamics: A hybrid approach

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## Abstract

This paper introduces a hybrid automata model for testing ideas and hypotheses relating to urban gentrification dynamics. We focus on the agency of relocating households in dynamic property markets as the theoretical basis for construction of the model. The methodology employed makes use of hybridized cellular- and agent-automata that allow for representation of co-interaction among fixed and mobile entities in urban settings across multiple scales. Simulations run with the model are based on various hypotheses from gentrification theory and these hypotheses are tested in simulation by running the model through theory-informed scenarios. The usefulness of this scheme is demonstrated through application of the model to a historically under-invested area of Salt Lake City in Utah that is undergoing recent transformation. Our results show that the hybrid approach is useful in representing human behavior in complex adaptive urban systems. Moreover, our model proves to be a useful test-bed for studying gentrification.

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## 1. Introduction

Gentrification refers to the transition of property markets from relatively low value platforms to higher value platforms under the influence of redevelopment and influx of higher-income residents, often with spatial displacement of original residents and an

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associated shift in the demographic, social, and cultural fabric of neighborhoods under its influence. The phenomenon has enjoyed the spotlight as a topic of academic inquiry in economics, politics, and sociology for about four decades. Two mainstream ideas predominate in the geographical literature: humanistic and Marxist approaches. Hamnett (1991) summarizes the distinction between the two in terms of the difference between “the liberal humanists who stress the key role of choice, culture, consumption and consumer demand, and the structural Marxists who stress the role of capital, class, production and supply.” (p. 174). Both approaches have been considered in urban geography contexts (Bondi, 1999; Cambridge Systematics & Group, 1991; Clark, 1992; Hamnett, 1991, 1992; Lees, 2000; Ley, 1987; Smith, 1987, 1992). There is general consensus, however, that the humanist and Marxist perspectives offer relatively translucent views of gentrification in isolation (Hamnett, 1991). An integrated explanation is needed, one that accommodates supply factors (the production of devalued areas and housing) and demand factors (the production of gentrifiers and their specific consumption and reproduction patterns).

Within geography, gentrification studies have largely been approached through theory-based consideration of causes and consequences. The topic remains largely untouched by modeling or simulation, despite the existence of a theoretical foundation for model-building and the need for synthetic simulation environments for exploring ideas that might not otherwise be open to investigation on the ground. There are barriers to entry, however. The breadth of the explanatory landscape across socioeconomic, cultural, political, and spatial factors often gets in the way of methodology. There is also a variety of scales of observation, as well as a diversity of agents and relevant factors, to be considered when model-building. Moreover, models are, by nature, data-hungry, and there is often little in the way to feed them when it comes to considering gentrification.

Treating gentrification as a complex adaptive system can help. The study of complex systems represents a relatively new approach to social science. Put succinctly, complex systems ideas focus on how the minutia of a system interact and co-adapt, often non-linearly, with each other and their environment, and how these dynamics might give rise to collective system-level phenomena. Traditionally, social science research has been challenged by a dichotomy between the individual and the aggregate. There are well-known barriers to overcoming this; ecological fallacy (Wrigley, Holt, Steel, & Tranmer, 1996) and the Modifiable Areal Unit Problem (Openshaw, 1983) are examples, as are difficulties in reconciling top-down reductionism and bottom-up generative science (Schelling, 1978).

The tool *par excellence* of complexity modeling—the automaton—can be used to overcome some of these difficulties. Automata have been used, successfully, to model a wide variety of complex urban phenomena (Benenson & Torrens, 2004): urban growth (Batty, 1991), land-use change (Engelen, White, Uljee, & Drazan, 1995), pedestrian dynamics (Haklay, O’Sullivan, Thurstain-Goodwin, & Schelhorn, 2001), residential mobility (Benenson, 1998), socio-spatial segregation (Portugali, Benenson, & Omer, 1997), vehicle traffic (Nagel & Rickert, 2001), and so forth. We are aware of a single gentrification application by O’Sullivan (2002), designed to test the use of graph-based formulations in human geography contexts. Automata are useful in these endeavors because of their ability to accommodate complex adaptive systems in a bottom-up fashion, catering to description of individual-level and heterogeneous dynamics at the micro-scale, but they are equally adept at interfacing with system-level considerations.

Use of cellular automata (CA) and multi-agent systems (MAS) separately in models of urban systems is commonplace, with the selection of one tool over another dictating the

sorts of questions that can be posed in simulation. There is general recognition of shortcomings to solely CA- or MAS-based approaches in urban simulation. CA are immobile in their simulated environments because individual automata are not free to move in the space in which they reside and also all spatial movement takes place through the diffusion of information through a neighborhood (Faith, 1998; Torrens, 2003, 2004; Torrens & Benenson, 2005). This makes them useful for representing landscapes and infrastructure, but not mobile entities. On the other hand, MAS are mobile and may be programmed with the freedom for true spatial mobility within the environments that they inhabit. This makes them useful for representing mobile entities, but rules designed to model movement and heterogeneous human agency are not always suitable for representing infrastructure.

The objectives of this paper are several-fold. An interest in constructing a simulation environment for studying urban gentrification is foremost in these objectives. Our goal is to build a system that can be used to explore both supply- and demand-oriented considerations, as well as unified approaches. We believe a complexity approach can be useful in unifying these views. We also believe that an automata-based methodology can be successfully applied in these endeavors, but we recognize the need for a hybrid scheme that leverages the advantages of both CA and MAS. These objectives cannot be realized without application to a real-world example.

Considering these goals, we have developed a model of inner city gentrification based on a theoretical foundation that caters to supply and demand determinants in a unified fashion. The modeling methodology makes use of a hybrid cellular- and agent-automata scheme, designed on a behavioral basis such that relocating households are represented, individually and collectively, through their interactions with the property market and its related social and economic landscape. Behavior is formulated based on a decision-making regime for households amid dynamic change. The model is put to the test in application to a formerly deprived area of Salt Lake City that is undergoing dramatic change, and simulations are run under theoretical scenarios designed to test gentrification hypotheses.

The paper is organized as follows. The model is described in Section 2, where the design of hybrid automata is discussed, the timeline for simulation is introduced, and the behavioral foundation of the model is described in the context of its theoretical foundation and mathematical formulation. The task of applying the model to a real world example is discussed in Section 3 with attention to the data resources employed and associated dataware design. Our simulation assumptions are declared in Section 3 and the initial seed conditions for simulation runs are detailed. Simulation scenarios are introduced in Section 4 and the results of those scenarios in simulation are discussed. The paper draws to a close in Section 5 with concluding remarks.

## 2. Model description

### 2.1. *The automata skeleton: entity types, characteristics, and nesting*

Our gentrification model makes use of a hybrid approach that combines agent automata and cellular automata but distinguishes between them on a behavioral level. Two types of automata are included: fixed automata, which act much like CA and are used to model properties and collections of properties; and mobile automata, akin to MAS and employed to animate residential households through urban (social, economic, property, community) space.

Fixed automata are considered, at the most micro-scale, over a regular lattice of square cells. We sacrifice some realism here; in reality, real estate boundaries are more likely to be irregular in delineation. There are advantages, however: the cell size is small enough (25 m) to represent individual buildings and properties. This approach also necessitates some generalization, as it is difficult to accommodate diverse sizes of real estate in a regular and uniform cell grid. We get around this problem by treating size as a state attribute of fixed automata. Each cell is endowed with a land parcel size value, and the model simulation considers this value rather than the raster (pixel) cell size. This preserves spatial relationships and tractability while allowing for the introduction of realism.

Geographically (and behaviorally), we specify three automata objects belonging to a larger **Fixed** super-class: `Market`, `Property`, and `FixedLand`. We are interested in micro-level dynamics, but also in macro-scale phenomena. To facilitate flow of information across scales and to allow interactions to occur on multiple levels, we nest these automata. Nesting allows state attributes to be collated across scales and made available as input to the decisions of other automata across scales (Benenson & Torrens, 2005; Torrens, 2006, in press; Torrens & Benenson, 2005). `Market` automata objects are used to capture meso-scale conditions and are formed as an aggregation of smaller-scale automata composed within them. The relationship is behavioral as well as structural. Automata may interact *within* a `Market` at that level; they may also react *to* the `Market`, itself, as an independent automaton. In this way, micro-scale interactions may take place and these changes can be updated dynamically at `Market` level.

`Property` and `FixedLand` automata objects operate within `Market` automata. The major distinction between the two is that `Property` automata are active; their state variables are malleable under the influence of transition rules. `FixedLand` automata, however, are passive in simulation. Their state conditions are immune to transition rules, but those states can factor into the decision regimes of other automata in the model. We consider two types of `FixedLand` automata: road and access point, which are used to introduce the influence of accessibility to sites outside the simulated system on residential behavior. Four access points are considered: downtown, highway entrance/exit, shopping mall, and grocery.

**Mobile** automata are introduced as `Resident` automata objects and are used to model households, their activities in `Market` objects, and their interactions with `Property` objects and other `Resident` automata. `Resident` automata are endowed with a set of state variables, including economic status, ethnicity, preferences for housing choice, and with the ability to sense their neighboring environment (beyond the fixed and symmetric neighborhoods classically employed in urban CA models).

The organization of automata in code is illustrated in Fig. 1; state variables (and their abbreviations in the following equations) for each automaton are listed in Table 1.

State variables that have a value range from 0 to 1 are normalized. They are converted from actual values to a 0–1 scale using the following formula:

$$\frac{V_i - V_{\min}}{V_{\max} - V_{\min}} \quad (i)$$

$V_i$  is an actual state value for automaton  $i$  (e.g., property price, property size, accessibility, or economic status (annual income)).  $V_{\min}$  is an actual minimum value across all automata, and  $V_{\max}$  is an actual maximum value across all automata.



Fig. 1. Structural hierarchy of modeled automata entities.

## 2.2. Event timeline for simulation

The simulation process is illustrated in Fig. 2. A number of events take place in simulation; taken together these events transition modeled households through their lifecycle and through the property market (Fig. 3). These activities in turn shape dynamics at

Table 1  
Automata state variables

Automata class	State variable
Market	Total number of properties
	Total number of residents
	Median property value
	Median residents' economic status
	Median accessibility to downtown (0–1)
	Median accessibility to highway (0–1)
	Median accessibility to mall (0–1)
	Median accessibility to grocery (0–1)
Vacancy rate (%)	
Property	Property price
	Property value (0–1)
	Property size
	Property size value (0–1)
	Land use (Vacant, Residential, Commercial, Industrial)
	Housing type (Single house, Duplex, Condominium, 3–4 units apartment 5–9 units apartment, 10 or more units apartment)
	Tenure (rent/own)
	Household capacity
	Number of occupied or rented residents
	Vacancy rate (0–100%: $V = O/Cp * 100$ )
	Accessibility to downtown (0–1)
	Accessibility to highway (0–1)
	Accessibility to mall (0–1)
	Accessibility to grocery (0–1)
	Neighborhood median property value (0–1)
Neighborhood median residents' economic status (0–1)	
Neighborhood ethnic rate	
Resident	Economic status (0–1)
	Ethnic status
	Settled status (Stay, Move)
	Resident's preferences
	Probability for a resident $i$ choosing a property $j$
	Probability for a resident $i$ leaving $i$ 's property $j$
	Threshold for resident $i$ 's probability of choosing a property
	Threshold for resident $i$ 's probability of leaving $i$ 's property

$i$  refers to a number of resident's identity: 1 to  $n$ ;  $j$  refers to a number of property's identity: 1 to  $n$ ;  $k$  refers to a number of ethnic identity: 1 to  $n$ ;  $l$  refers to a number of market: 1 to  $n$ .

the level of sub-markets and the entire urban area, which have an influence again on the micro-scale. In this way, gentrification is allowed to “emerge”—we use the term cautiously (Faith, 1998)—through the actions and interactions of automata across scales.

The simulation begins with parameterization of automata entities. Initial state conditions are mapped onto the state variables of fixed and mobile automata at the atomic (in terms of modifiable areal units) level of Property and Resident automata. These states are aggregated to create variables for Market automata.

Time proceeds in simulation as packets of change, with distinct simulation processes taking place within discrete bundles of time. At each time step, four main processes are simulated: decisions of current residents regarding whether to move or stay *in situ*, inflow

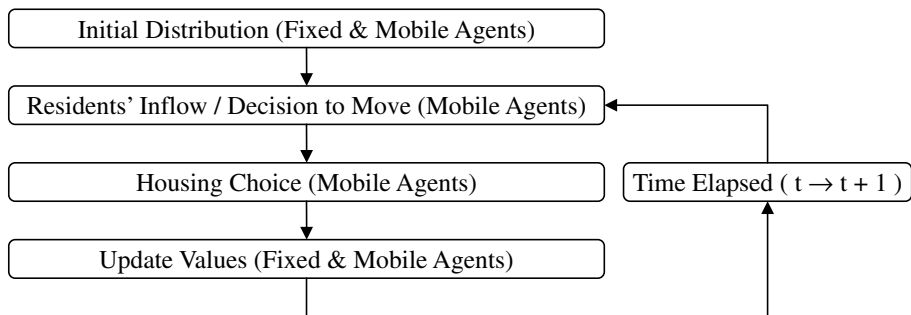


Fig. 2. The temporal flow of events in simulation.

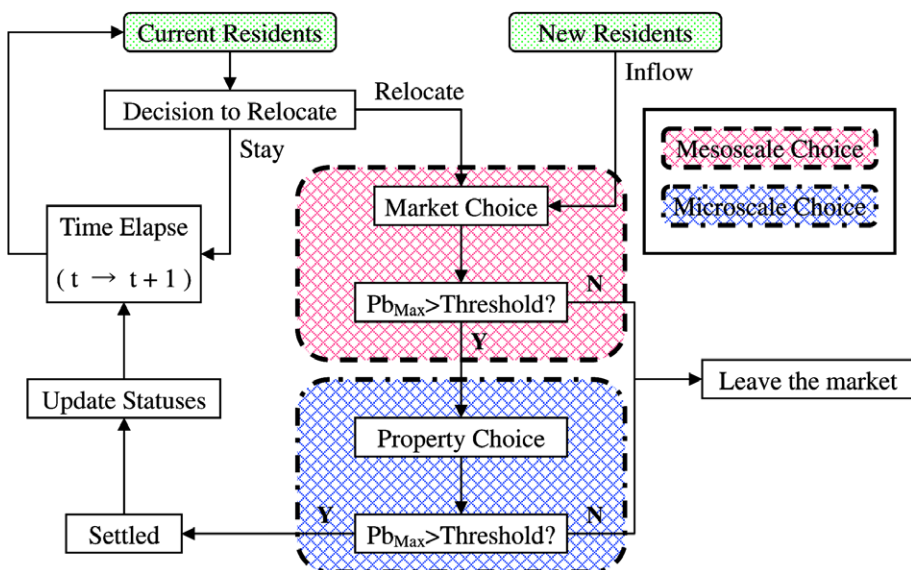


Fig. 3. The residential housing choice process.

of new residents to the area, housing choice, and updating of state variables. This follows schemes outlined in theory by Rossi (1955) and Clark (1993). The first three of these events are governed by interaction among simulated entities through exchange of state information and processing of that information using transition rules.

*Decision to move*—Under this event, current residents in the urban area calculate a likelihood that they will move. This is the seed event for future search dynamics.

*Inflow of new residents*—This process introduces new residents to the urban area, as consumers of real estate. They have already settled upon a relocation event and will engage in a housing search within the market. If that search is successful they will be added to the existing population and may displace an existing resident. If the search is unsuccessful, the searcher will exit from further consideration.

*Housing search*—A dedicated search regime is initiated if a household (an existing resident who wishes to relocate, or newly incoming households who are interested in

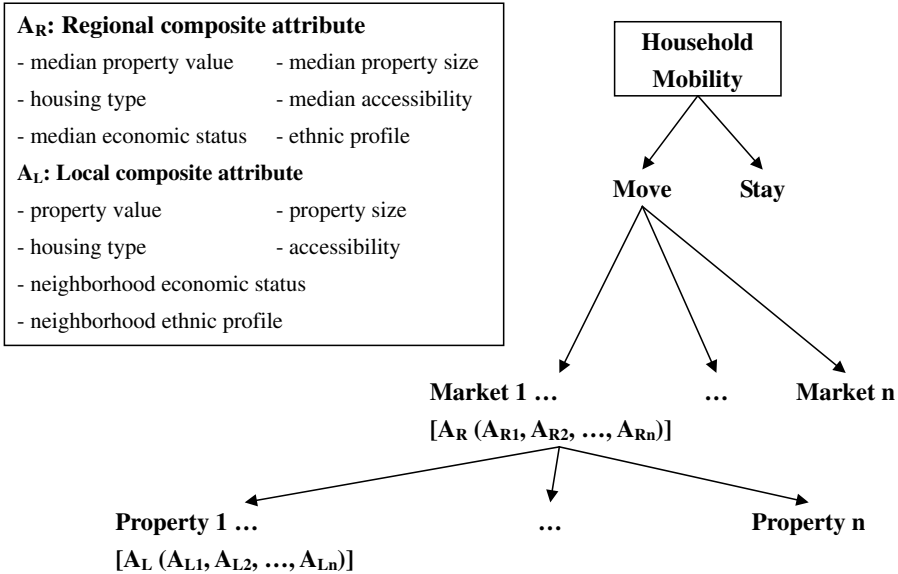


Fig. 4. Hierarchical nested tree of household mobility, macro- and micro-choice.

the market) has decided to engage in a real estate search. We make use of the concept of hierarchical nested choice in formulating these events in simulation. The decision-making process regarding real estate can be described by choice at two hierarchical levels: regional and local. At a regional level, the choice is among spatial clusters (property sub-markets, communities) as discrete alternatives at that scale of geography. At a local level, residents are faced with decisions about individual properties as discrete alternatives. The two scales are nested—choice of a cluster at a regional level locks the resident into considering only alternatives within that cluster (Fig. 4). Agents are not exposed to higher-level temporal dynamics until their activity in that lower-level geography has been exhausted (and this time period is not fixed; agents can terminate a search at any time). A volume of evidential and theoretical literature exist to suggest that households organize their search hierarchically in this fashion (Clark, 1982a, 1982b, 1993; Clark & Flowerdew, 1982).

### 2.3. Behavioral specification

The specification of modeled entities into fixed and mobile types, and their nesting across scales, sets the stage for construction of relatively realistic gentrification simulations. The hybrid automata approach facilitates this, but the real advantage of using automata lies in their ability to support independent, individual, heterogeneous treatment of interacting entities and the dynamic systems they form and are influenced by. This can only be achieved with attention to the *behavior* of modeled entities.

Our model is anchored to a rich theoretical foundation. Much of this foundation is based on household behavior in real estate markets, although we also consider behavior in, around, and relating to property and community.



### 2.3.1. *Lifecycle transition*

Households pass through a lifecycle in the model: they interact endogenously and with the clock in simulation, independently of other influences. In this sense, their state variables transition internally without neighborhood influence. This is akin to treatment of state transition in Markov models and its role in residential mobility is well-described in the theory literature.

### 2.3.2. *Propensity for mobility*

Propensity for mobility is a key behavioral component in the model. Without it, households succumb to inertia and do not move. The likelihood of mobility is based on endogenous factors (dynamically adjusted on the basis of lifecycle transition) and exogenous factors in the model. This follows Clark and Cadwallar (1973) in theoretical foundation and McFadden (1974) in methodological specification.

### 2.3.3. *Discrete decision-making*

We make use of a discrete choice approach in modeling household decision-making. Available alternatives (property sub-markets, real estate units) are regarded as discrete for decision-making purposes. The decision threshold is based on a utility-like calculation that weighs the household-specific benefits to be obtained from available alternatives. Decisions are not universal across all modeled households; households filter their evaluations through independent preferences. An element of chance is introduced: utility of alternatives is expressed in terms of likelihood of choice relative to other alternatives. This introduces the notion that households *may not always act rationally*.

Choice is hierarchical in nature in the model, with geography as the bifurcating mechanism. Households are presented with alternatives at two scales: the sub-market/community level and at the level of individual units of real estate. Theory suggests that these two scales are mutually inclusive (van der Vlist, Gorter, Nijkamp, & Rietveld, 2002). Households nest their decision-making across these scales. First, they choose among spatial clusters in the form of property sub-markets. Once a cluster has been selected, the search for a property is *focused* within that cluster. There is an element of path-dependence to decision-making in that the decision-making process is locked-in to a given cluster. There is, however, evidence that search is stratified in this way in the real world (Clark, 1982b).

### 2.3.4. *Real estate “behavior”*

Real estate does not really have any behavior. Houses do not put themselves on the market and they do not move; people offer them for sale and relocate. Real estate dynamics are a by-product of the actions of the people that populate, consume, evaluate, and value them. Any behavior that we might associate with property is a reflection of human activity; real estate is the vessel for this activity.

We represent activity in and around real estate in our model by considering the interface between households and properties in a hedonic fashion. Put succinctly, we consider value (empirically) as a bundle of its attributes or characteristics. For example, real estate price can be considered as a bundle of attribute prices assigned to its land footprint, the external structure, floor–area ratio, location, and neighborhood externalities. Attribute values are commonly referred to as an implicit price or a hedonic price, which leaves us with a hedonic approach to determining overall price as a function of implicit prices (Hidano, 2003; Rosen, 1974).

The treatment of real estate price also warrants mention. In the real world, the price is set by the household in conjunction with their realtor and in consideration of their expectation of what the market will pay. Needless to say, this is quite difficult to model when mapped to individual households in simulation. We approach price in a proxy manner, with the assumption that price is driven by vacancy rates. This has justification in the theoretical literature (DiPasquale & Wheaton, 1996) and is popularly used in urban economic models (Waddell, 2000). A vacancy adjustment function updates property value through examination of market-level vacancy rates. In this sense, we have represented a very abstract market mechanism that accounts for inequity between supply and demand.

### 2.3.5. Self-organization

Residential clusters are also understood to self-organize in space and time (Allen, 1997; Krugman, 1996; Portugali, 2000). This is a by-product of how they are built: high-value real estate tends to be constructed in agglomerations (Chicago's Gold Coast, Phoenix's Paradise Valley, Beverly Hills in Los Angeles). Other aspects of real estate self-organization relate to collective behavior, however. Socio-spatial segregation based on ethnicity is an example (Schelling, 1978).

We are interested in gentrification as an example of self-organization. The nesting of modeled entities allows for self-organization to propagate across scales and for phenomena that emerge from micro-level interactions to be caught as novel forms at meso- and macro-scales.

## 2.4. Model formulation

The model was implemented in NetLogo and constructed based on the aforementioned design. Specifically, hedonic valuation, propensity for mobility, nested choice, accessibility, preference-based mediation of choice, and price adjustment are formulated as follows.

### 2.4.1. Hedonic valuation

Valuation of property is considered hedonically, such that

$$P_j = C + \sum_{k=1}^n V_k Q_k \quad (\text{ii})$$

Above,  $P_j$  refers to price of real estate unit  $j$ .  $k$  refers to attributes of that property bundle,  $k = 1, 2, \dots, n$ .  $V_k$  represents the value of attributes  $k$ , while  $Q_k$  represents the quantity of attributes  $k$ .  $C$  is a constant.

### 2.4.2. Propensity for mobility

Likelihood of relocation is calculated as follows:

$$Pb_{Lij} = 1 - Pb_{Cij} \quad (\text{iii})$$

Above,  $Pb_{Lij}$  describes the likelihood that household  $i$  will leave property  $j$ .  $Pb_{Cij}$  indicates how much household  $i$  prefers its property  $j$ , and  $(1 - Pb_{Lij})$  indicates how much a household does *not* prefer that property.

### 2.4.3. Nested spatial choice

Spatial choice is formulated in the following manner.

$$Pb_{Cij} = \sum(b_{HE} \cdot H_E) + \sum(b_{NE} \cdot N_E) \tag{iv}$$

Above,  $Pb_{Cij}$  indicates the likelihood of household  $i$  choosing property  $j$  from a set of discrete alternatives.  $b_{HE}$  is a weighting for property;  $b_{NE}$  is a weighting for markets.  $H_E$  is a set of property characteristics (a bundle of attributes);  $N_E$  is a set of market characteristics. Eq. (iv) can be expanded, and this illustrates the incorporation of preferences in the calculation of likelihood of choice.

We can expand choice at the property level ( $\sum(b_{HE} \cdot H_E)$ ).

$$\begin{aligned} \sum(b_{HE} \cdot H_E) = & b_1(1 - |P_{Vj} - E_{Si}|) + b_2(H_{TSj} \cdot R_{PTSi} + H_{TCj} \cdot R_{PTCi}) \\ & + b_3(1 - |P_{Sj} - R_{PSi}|) + b_4 \sum(A_j \cdot R_{PAi}) \end{aligned} \tag{v}$$

Eq. (v) includes factors relating to the alternative being evaluated, as well as characteristics of the household doing the evaluating. Characteristics of the alternative are considered hedonically, and are treated endogenously and exogenously.

The  $b_m$  values in Eq. (v) are weights, used to increase or reduce the relative importance of other factors on likelihood of choice. These weights are scaled across Eqs. (v) and (vii) such that  $\sum_1^6 b_m = 1$ . The weights should be interpreted as follows: coefficients for property value suitability ( $b_1$ ), house type preference ( $b_2$ ), house size preference ( $b_3$ ), accessibility preference ( $b_4$ ), neighborhood economic status ( $b_5$ ), and neighborhood ethnicity ( $b_6$ ). Agent-specific characteristics are represented by economic status ( $E_{Si}$ ), and a series of  $R$ -values that represent residents' preferences for property characteristics ( $R_{PTSi}$ : resident  $i$ 's preference for single-family housing;  $R_{PTCi}$ : resident  $i$ 's preference for condominiums;  $R_{PSi}$ : resident  $i$ 's property size preference;  $R_{PAi}$ : resident  $i$ 's accessibility preference). Endogenous characteristics include property price ( $P_{Vj}$ ), property type ( $H_{TSj}$  is a dummy variable for house;  $H_{TCj}$  is a dummy variable for condominium), and property size ( $P_{Sj}$ ).  $A_j$  is a variable for  $j$ 's accessibility.

### 2.4.4. The influence of accessibility

Exogenous factors enter into consideration through accessibility ( $A_j$ ). Accessibility is treated in terms of access to four locations: downtown ( $A_{DTj}$ ); the nearest highway entrance/exit ramp ( $A_{HWj}$ ); the nearest shopping mall ( $A_{Mj}$ ); and the nearest grocery store ( $A_{Gj}$ ).

$$\begin{aligned} \sum(A_j \cdot R_{PAi}) = & (A_{DTj} \cdot R_{PADTj} + A_{HWj} \cdot R_{PADHWj} + A_{Mj} \cdot R_{PAMj} + A_{Gj} \cdot R_{PAGj}), \\ \text{where } \sum R_{PAi} = & 1 \end{aligned} \tag{vi}$$

Following McFadden (1974), we also consider characteristics of the household doing the choosing: economic status of household  $i$  ( $E_{Si}$ ).

### 2.4.5. Preference-mediated choice

The  $R$ -values in Eqs. (v) and (vi) are preferences. These act as a filter through which decisions are made and they serve to tailor the decision calculation to independent households. These preferences are introduced at the level of the alternative being considered (preference for houses ( $R_{PTS}$ ); condominiums ( $R_{PTC}$ ); and house size ( $R_{PS}$ )), as well as

exogenously through accessibility ( $R_{PA_i}$ , decomposed to preferences for accessibility to downtown ( $R_{DT_i}$ ), highways ( $R_{HW_i}$ ), shopping malls ( $R_{M_i}$ ), and grocery stores ( $R_{G_i}$ )).

We can also expand choice at the market-level ( $\sum(b_{NE} \cdot N_E)$ ), such that

$$\sum(b_{NE} \cdot N_E) = b_5(1 - |E_{SM_j} - E_{S_i}|) + b_6 \cdot E_{R_i} \quad (\text{vii})$$

The  $b$ -values are weights, as before, but in this case they are used to weight market-level conditions: market-wide economic status ( $b_5$ ) and ethnicity ( $b_6$ ). Taken together, the weights across Eqs. (v) and (vii) sum to unity;  $\sum_1^6 b_m = 1$ . The  $E$ -values are used to introduce market-level attributes to households' likelihood calculation: the median economic status of the market that real estate unit  $j$  is associated with ( $E_{SM_j}$ ) and market-level ethnicity ( $E_{R_i}$ ).

#### 2.4.6. Price adjustment

Finally, the vacancy adjustment function for property price is formulated as follows:

$$P_V(t+1) = P_V(t) \cdot \left[ \frac{1 + \alpha_b - V_{bl}(t) + \lambda \cdot (1 + \alpha_b - V_{bl}(t))}{1 + \lambda} \right]^\beta \quad (\text{viii})$$

Above,  $P_V(t+1)$  represents the price of a real estate unit in time  $t+1$ . This price is a function of a combination of its price in the previous time step  $t$ , the vacancy rate ( $V_{bl}(t)$ ) for space in real estate type  $b$  in location  $l$  at time  $t$ ; the normal vacancy rate ( $\alpha_b$ ) for real estate type  $b$ ; a scaling parameter for the property price adjustment ( $\beta$ ), which is initially set to a value of 1; and a parameter for weighting system-wide influence ( $\lambda$ ).

### 3. Applying the model to a real-world scenario

We applied this general behavioral model to a real-world example: the Gateway district of Salt Lake City, Utah and its surroundings. The area encompasses a formerly under-invested and largely industrial part of the city's downtown (Fig. 5). The Gateway district is subject to a long-term development plan to convert the former zone in transition to residential and retail uses with a multi-mode transport hub several years into the future. The core of this area is a former warehouse-dominated district that has fallen on hard times. The surrounding area is dominated by residential uses: single-family and multiple-occupancy residential and is home for an ethnically diverse minority of the downtown's population, but has traditionally been under-funded. Visually, the area appears to be in early stages of gentrification, with a newly burgeoning artist population, a fledgling graphic design and new media industry, and planned loft-style, live-work-play, condominiums. Gentrification is happening in the area; the main question that we would like to pose with the model are what mechanisms are driving gentrification and what likely futures for the area are on the horizon under different scenarios?

#### 3.1. Dataware

The Gateway area plays host to some of Salt Lake City's most dynamic urbanization. Because of development plans, a wealth of data resources exists with which we can build an application of our gentrification model. As is usually the case in these situations, we are relatively data-rich with respect to some variables of interest, but data-poor with respect to

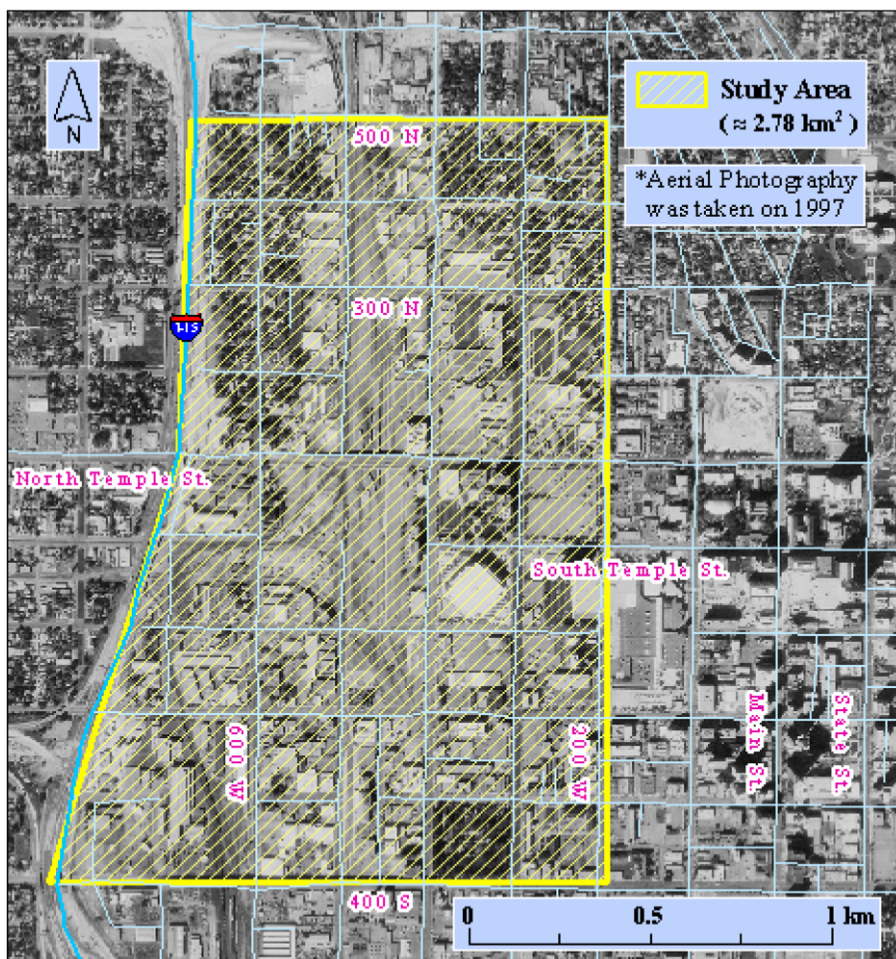


Fig. 5. Study area: Salt Lake City's Gateway district.

others. A list of data resources used to apply the model is presented in Table 2. A limited set of property data are at parcel scale, suitable for direct inclusion in the model. However, resident data are not at household scale; rather, they are available at regional scale. We constructed a synthetic data population to reconcile the two (Bush, 2001). Household-level data were estimated from higher-scale data, or assigned random values, in such a way that the totals at micro-scales matched known totals at larger scales (Census blocks and blockgroups).

### 3.2. Simulation assumptions

The simulation applied to the study area is based on some underlying assumptions. First, we do not consider land-use transition. Second, we only consider owner-occupied properties. Third, we make a distinction between single-family homes and condominiums

Table 2  
Data resources

Data type	Year	Scale	Resources
<i>Property</i>			
Value (\$)	2004	Parcel	Salt Lake County Assessor's office
Size (ft <sup>2</sup> )	2004	Parcel	Salt Lake County Assessor's office
Type	2004	Parcel	Salt Lake County Assessor's office
Location	2004	Parcel	Salt Lake County
<i>Road network</i>	2000	1:100,000	The Census 2000 TIGER/Line data
<i>Resident</i>			
	2000	Census block	The Census 2000 Summary File 1
	2000	Census block group	The Census 2000 Summary File 3
	1990	Census block group	The Census 1990 Summary File 3

in describing real estate types. Fourth, ethnicity is considered as Latino and non-Latino. Fifth we use maximum utility as the threshold mechanism for choice.

The first four of these assumptions are due to data limitations, but also help with model simplification. We consider only residential properties, and the process of land-use transition is eliminated as the main aim is not to represent land-use transitions but to represent gentrification phenomena. Residents are characterized as either non-Latino or Latino because these are the two major ethnicities in the study area.

### 3.3. Initial simulation conditions

Individual residents are initialized with an economic status that reflects the value of their property, with some random perturbation (Table 3). Other characteristics (preferences, weights) are assigned randomly in the absence of survey data.

Table 3  
Initial parameter settings for each scenario

Category	Parameter	Definition	Scenario			
			1	2	3	4
Population	$G_{POP}$	Population growth	10	10	12	12
	$D_{POP}$	Population decrease	2	2	2	2
Residents	$N_{CELL}$	Neighborhood cell	10	10	10	10
Property	$V_{Rm1}$	Vacancy rate for Market 1	0.15	0.15	0.15	0.15
	$V_{Rm2}$	Vacancy rate for Market 2	0.15	0.15	0.15	0.15
	$V_{Rm3}$	Vacancy rate for Market 3	0.15	0.15	0.15	0.15
Vacancy adjustment function	$\alpha_S$	Normal vacancy rate for single house unit	0.15	0.15	0.15	0.15
	$\alpha_C$	Normal vacancy rate for condominium unit	0.15	0.15	0.15	0.15
	$B$	Scaling parameter	1.00	1.00	1.00	1.00
	$A$	Weight for regional and zonal influence	0.025	0.025	0.025	0.025
Gentrifier	$G_{nt}$	Gentrifier parameter	–	3.00	–	3.00

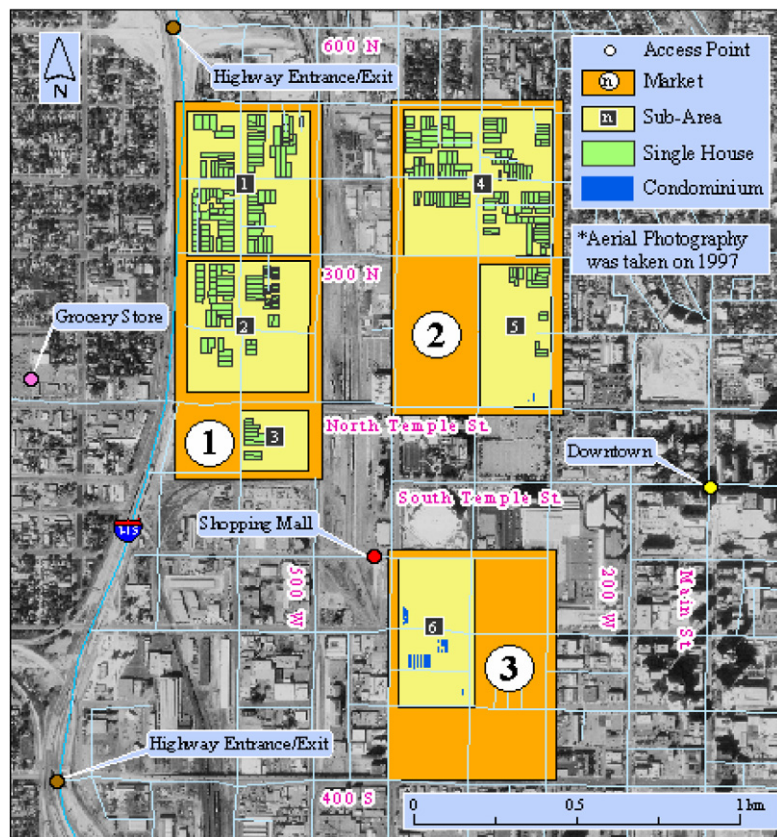


Fig. 6. Market boundaries and access points.

We define three markets upon initializing the model. These markets are based on reasonable delineation of property sub-markets for the area (Fig. 6). The markets have some distinct attributes. All of Market 1 and most of Market 2 are occupied by single-family housing units, although the market is divided (physically, socially, and economically), by the city's main railroad. By contrast, Market 3 is dominated by condominium units, and is advertised as a work-live-play community. The northern markets (Markets 1 and 2) are physically separated from the south market (Market 3) by a buffer of office and civic space. Markets 1 and 2 are also separated by a railroad. Socio-economic divisions are also present. Market 1 is home to a majority Latino community, Market 3 is mostly non-Latino in population, and Market 2 is mixed. Economically, Markets 1 and 2 are lower-income markets and Market 3 is a relatively high-value-platform market.

#### 4. Testing ideas in silico through simulation scenarios

We are interested in using the model to create synthetic, but realistic, laboratories for testing ideas relating to gentrification dynamics. We have run the simulation under different scenarios with this in mind. The goal is to test hypotheses about demand and supply

factors of gentrification, to examine how gentrifiers and gentrifiable properties act as drivers of gentrification dynamics, in isolation and in unison. Four scenarios are run.

*Base simulation*—The first scenario is the base simulation. This is a business-as-usual scenario. The parameters for simulations run under this scenario are described in Table 3. Population growth ( $G_{POP}$ ) is assigned a value of 3%; the average monthly household population growth from 1990 to 2000 is 3.217% in the study area. Other parameter values are empirically assigned since sufficient data are not available.

*Introducing potential gentrifiers*—The second scenario considers demand-side factors of gentrification theory. The scenario poses questions relating to the implications of potential gentrifier inflow to the study area. This is animated in simulation by raising the economic status of new residents.

*Introducing potential gentrifiable properties*—The third scenario considers supply-side factors of gentrification theory. Under this scenario, new supply is introduced between Markets 1 and 2 in the form of additional development (which we refer to as a new market, Market 4).

*A combined theory, gentrifiers and gentrifiable properties*—Finally, the fourth scenario considers both demand and supply-side theories. This scenario combines scenarios 1, 2, and 3.

#### 4.1. Results

The model is quite rich and a volume of results can be harvested from simulation runs. In order to examine our hypotheses, we will first focus on space–time dynamics of gentrification, relatively, across the four simulation scenarios. We detail dynamics in attribute-space, specifically, thereafter, concentrating on property value and economic status as well as ethnic mix and residential displacement.

##### 4.1.1. Gentrification dynamics under the four simulation scenarios

The model was run for 500 simulation time steps under each scenario and area-wide average values of total household, property value, economic status, original resident profile, and non-Latino ethnicity profile were collected dynamically. It is assumed that one simulation time step equals a month since the household population growth ( $G_{POP}$ ) is monthly growth; therefore, 500 simulation time steps correspond to approximately 40 years.

The geographical dynamics of the simulation under the demand–supply scenario is depicted in Fig. 7; the temporal dynamics across all scenarios are illustrated in Figs. 8–11. The supply-side scenario is not sufficient for explaining gentrification dynamics. The same can be said of the demand-side scenario and the base simulation (business as usual). Only the demand–supply scenario explains gentrification dynamics fully (Figs. 8–11).

The base scenario yielded some volatility across attribute-space over the course of the simulation run. The three markets remained distinct nevertheless. Markets 1 and 2 shared dominance as home to the majority of the area's population at six time points in the simulation, while Market 3 remained distinct, housing a minority. There was no visible correlation between these dynamics and transitions in other attributes. Property values were quite dynamic. Affluent Market 3 appreciated markedly early in the simulation runs, flattening-out thereafter. Growth in value was more steady in lower-income Markets 1 and 2. Market 2 actually eclipsed Market 3 in value for a brief period in the simulation.



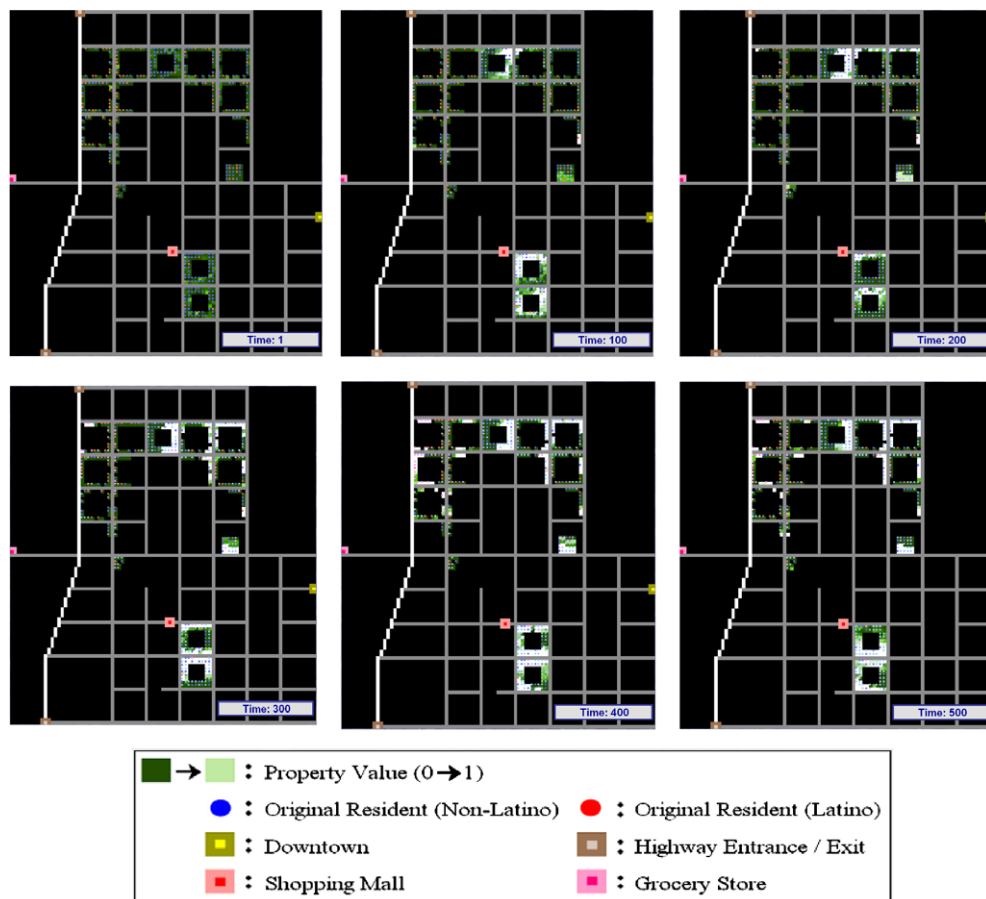


Fig. 7. Geographical dynamics under the demand–supply scenario.

Despite early tight-coupling, Markets 1 and 3 demonstrate almost reciprocal trends in value throughout the remainder of the simulation, with booms in “yuppie” Market 3 shadowing busts in lower-income Market 1 and vice-versa. Economic statuses remained stable over the simulation. Markets 1 and 2 stay lower-income, and Market 3 remains as a higher-income community, regardless of property price fluctuations. Ethnically, the markets remained distinct. Market 3 stayed non-Latino; Market 2 remained relatively mixed; and Market 1 remained Latino. There was, however, a trend of growing divergence between Markets 1 and 2. Market 1, in particular, grew more homogeneously Latino as the simulation progressed, while mixture in Market 2 increased slightly. The Latino population never really gained entry to Market 3, even at times when property values dropped. The simulation demonstrated a steady displacement of original residents over the course of the simulation, across markets.

The status quo is maintained for the most part over the course of the simulation under the demand-driven scenario. Whereas the base scenario was relatively mixed, the three real estate markets tend to grow increasingly distinct and entrenched in their characterization

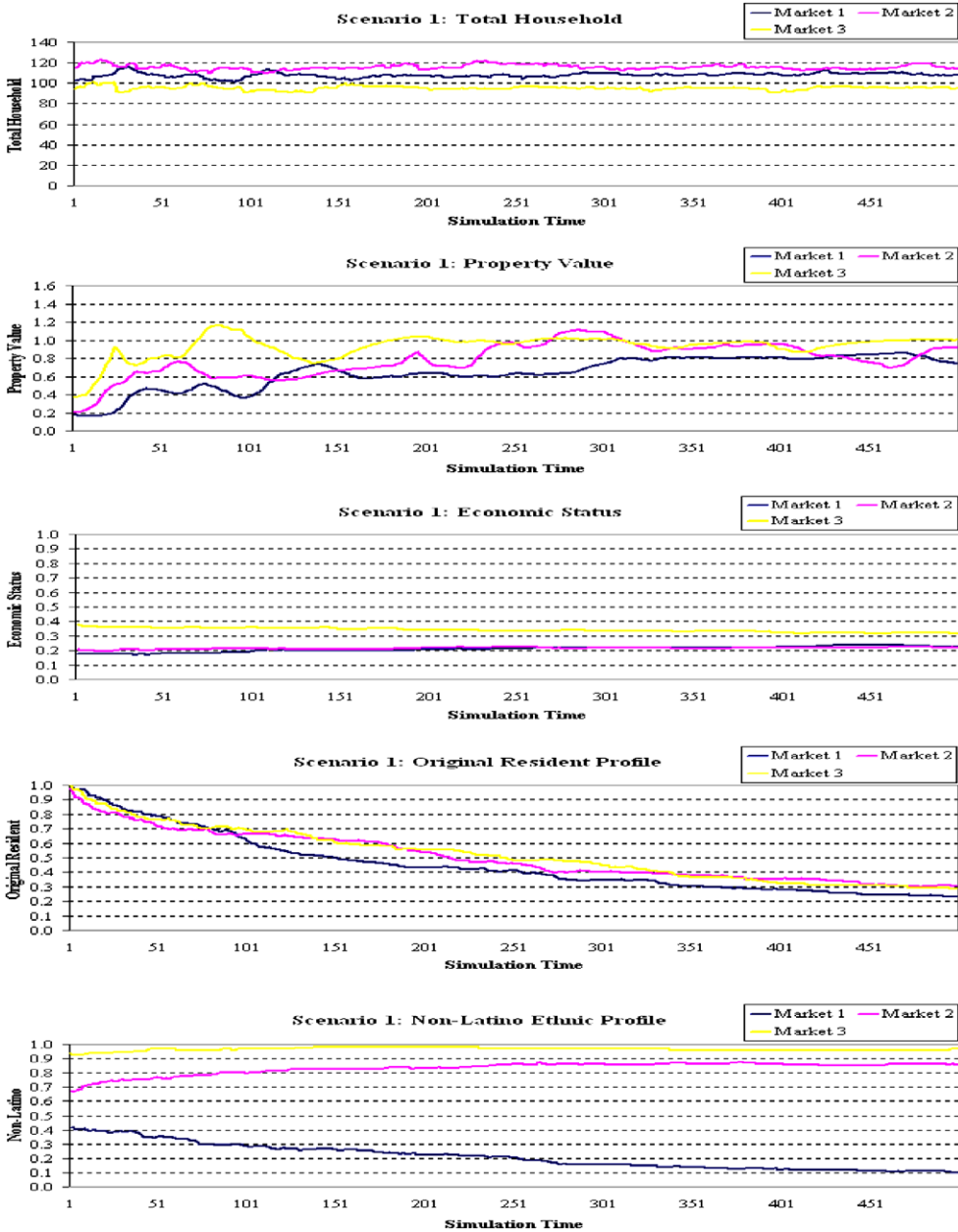


Fig. 8. Temporal dynamics under the base scenario.

under the demand scenario. Market 2 retains the majority of the area’s population throughout the simulation, as was the case on the base scenario. There is a great degree of volatility in total household count, however. The range of fluctuation ( $\pm 40$ ) was the greatest of all scenarios. Divergence across markets grew steadily over time. Separation

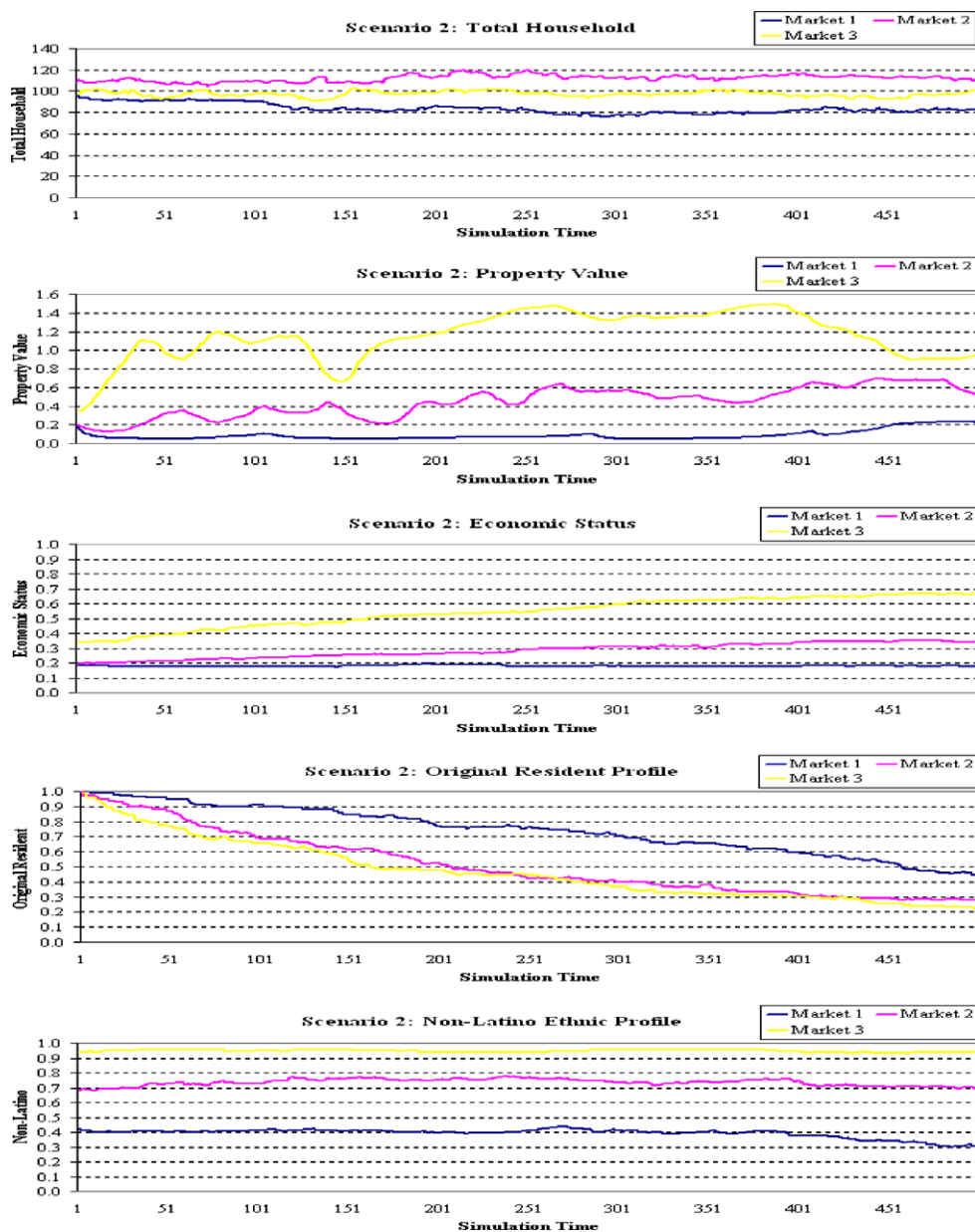


Fig. 9. Temporal dynamics under the demand scenario.

in property values across the markets is dramatic. “Yuppy” Market 3 diverges from lower-income Markets 1 and 2 in appreciation very early in the simulation, growing increasingly so until a period of bust toward the end of the simulation run. Market 2 appreciates relatively slowly, with a high degree of volatility, while Market 2’s value stays flat throughout the simulation. There is some reciprocity in the relationship between the fortunes of

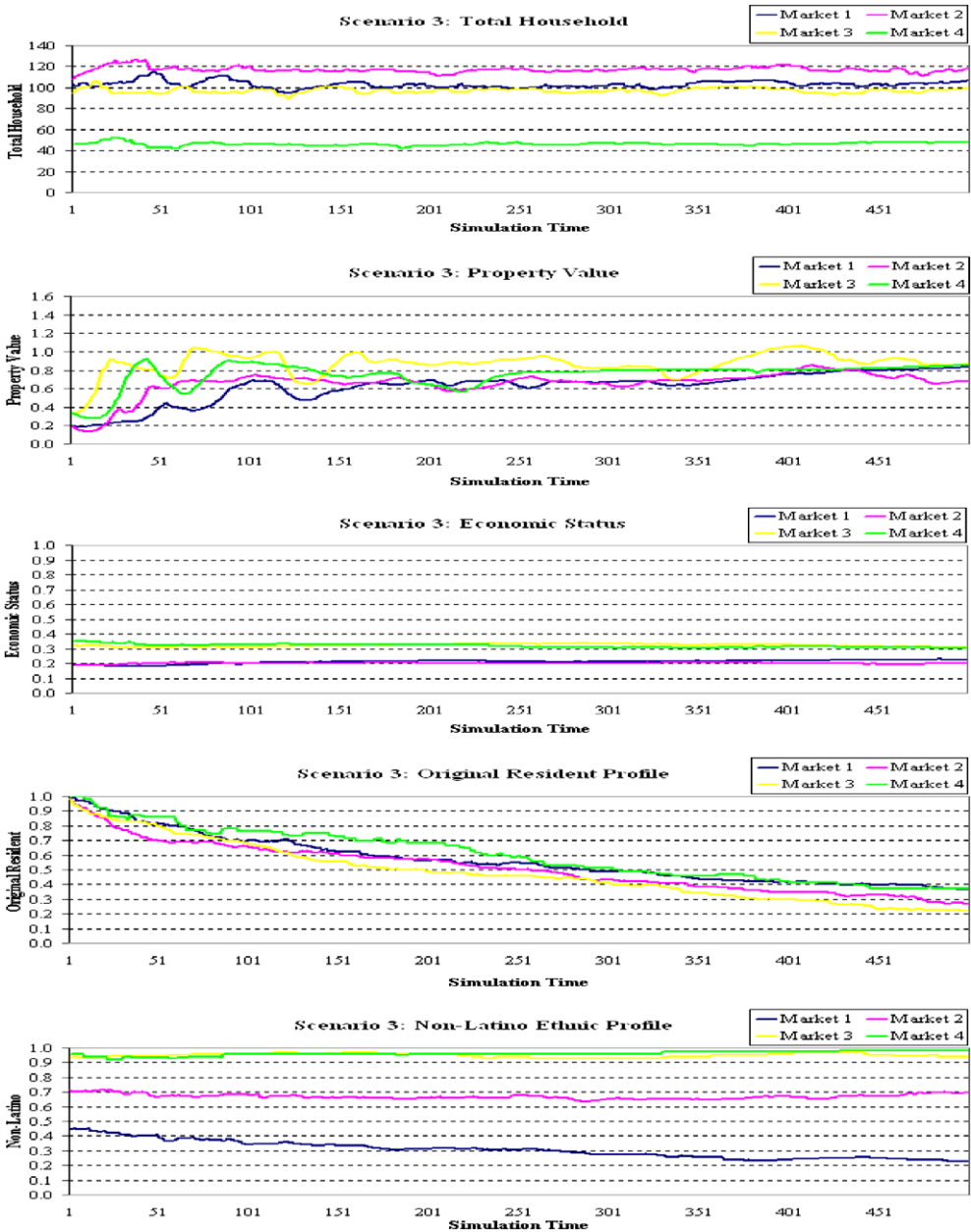


Fig. 10. Temporal dynamics under the supply scenario.

Markets 2 and 3, with bust trends in one mirroring boom trends in the other. Economic status and property value did not seem closely tied in the base scenario, but they are tightly coupled in the demand scenario. Economic status follows property value steadily across

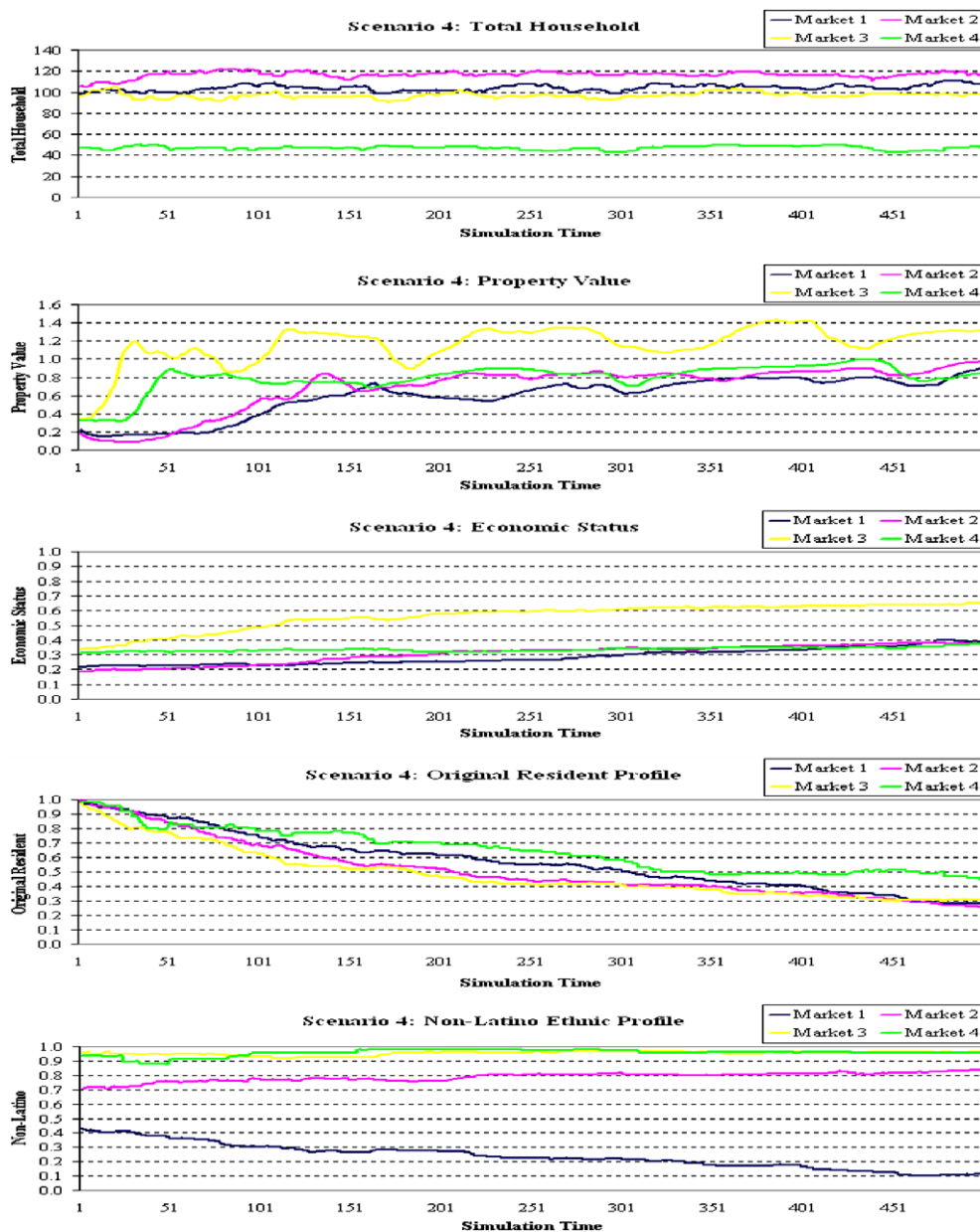


Fig. 11. Temporal dynamics under the demand–supply scenario.

markets and over the timeline of the simulation. There is a steady displacement of original residents from Markets 1 and 3, but the trend is less dramatic in Market 2. Ethno-spatial segregation is marked across markets, mimicking econo-spatial separation, but there is little fluctuation in the degree of relative separation, despite the volatility in property values.

The supply-driven scenario produces amazing synchronicity across attribute-space. Market 2 dominates in housing the majority of the area's population, as before. The newly added Market 4 absorbs only a small volume of the population and has almost no influence in drawing population away from the other markets. Property values are immediately volatile in the simulation run, flattening-out to a slow but steady growth thereafter. The newly introduced Market 4 has a competitive effect on "yuppy" Market 3. The real estate fortunes of the four markets are very closely intertwined, however, following the same boom and bust cycles after initial volatility early in the simulation. There are clear spatial distinctions in the economic status of residents: Markets 3 and 4 remain relatively affluent, while Markets 1 and 2 remain lower-income. This juxtaposition does not shift at all through the simulation. The trend mimics that of the base scenario almost exactly, but is slowly negative-tending, whereas that of the demand-driven scenario was strongly positive-tending. The decline in original residents is steady across markets, largely mimicking that of the demand-driven scenario, but with less divergence. The rate of decline in original residents is less dramatic than under the base scenario. Spatial separation remains marked across markets. Newly formed Market 4 is overwhelmingly non-Latino in character, and remains so over the simulation. Rates of change mimic those in the demand-driven scenario, but are quite different from the base scenario.

The combined, demand- and supply-driven scenario closely resembles that of the base scenario, across attribute-space. The rate of change in total household growth is relatively volatile, with a range of  $\pm 20$ . Economic status grows steadily, with marked spatial separation between "yuppy" Market 3 and lower-income Markets 1 and 2. Even as wealth grows, the markets remain distinct rather than evening-out. Original residents are displaced from all markets. Ethnically, Market 2 remains mixed, while Markets 1 and 3 remain homogeneously Latino and non-Latino respectively. Newly formed Market 4 is overwhelmingly non-Latino. There are some very interesting trends in property values, however. Value grows across all markets, steadily and dramatically over the simulation run. Condominium-dominated Market 3 shows marked volatility, but trends upward in value.

#### *4.1.2. Gentrification signatures in property value, economic status, ethnic profile, and resident displacement dynamics*

Property values are dynamic across all scenarios, both temporally and geographically. Introduction of gentrifiers alone drives prices up, and remarkably so, in Market 3 (which is dominated by condominium units that are marketed to young, up-and-coming, professionals) and this trend remains steadfast over time. The demand-side explains gentrification dynamics partially. The introduction of new supply in isolation has a stabilizing influence on price in the long-run, however. This is quite different to dynamics under the demand-only scenario.

Only the combined demand–supply scenario explains gentrification dynamics in property process successfully. The condominium-dominated market (Market 3) is elevated to a distinct value-platform, but the long-term trend is for prices to climb across the urban area. There is a spill-over, by diffusion of real estate price growth, into the formerly under-invested Markets 1 and 2.

The business-as-usual scenario holds economic status relatively stable over time, which is understandable. The introduction of new gentrifiers raises economic status markedly and this is particularly evident in the market that is designed to cater to this resident

profile. Market 2, which has been historically under-developed but enjoys all the benefits of accessibility that Market 3 has, also plays host to a dramatic rise in economic status and this creates spatial disconnect with neighboring Market 1. The introduction of supply alone has a stabilizing influence, with the result that economic status remains steady.

Once again, only a combined supply and demand scenario is sufficient in explaining system-wide economic response to gentrification. The economic geography of the urban area takes on some interesting characteristics under the demand–supply scenario. The introduction of a new market between Markets 2 and 3 provides an economic bridge between otherwise economically distinct clusters of real estate and a supply of gentrifiers to caravan the pattern between the two. The gentrification dynamics affect property values; they also impact the population.

The results for ethnic profile were less dramatic. The business-as-usual scenario maintains the status quo for ethnic profile over the urban area. Influx of potential gentrifiers has a stabilizing influence. The introduction of property supply alone did little to alter the balance. The demand–supply scenario did maintain a division on ethnic lines, however.

The influence on ethnic mix is less dramatic, largely because we do not have data for ethnicity of the 3% cohort of new population supply. Tilting the mix would likely produce dramatically different dynamics, but without a solid basis for tilting the parameters one way or another, the influence remains artifactual. We did, however, observe marked geographic dynamics. As with the economic influence of the demand–supply scenario, the introduction of new supply and new gentrifiers in Market 4 established a connection between Markets 3 and 4, which otherwise demonstrate socio-spatial segregation along ethnic lines.

Over time, the proportion of original residents remaining across markets (from time  $t = 1$  to time  $t = 500$ ) declines, as it should, under the business-as-usual scenario. This decline is accelerated under the introduction of new residents in the demand scenario, but remains steady under the other scenarios. Geographically, the supply and demand–supply scenarios produce some interesting results. The process of displacing original residents in Market 1 accelerates dramatically under the introduction of Market 4.

## 5. Conclusions

Gentrification, a term that was first coined by Ruth Glass in 1964, has been widely discussed in the field of urban geography. This literature base largely focuses on theoretical issues relating to causes and consequences of gentrification. There has, by contrast, been relatively little research into testing these ideas in simulation, despite the advantages that simulation can offer as a synthetic test-bed for hypotheses that are not easily explored on the ground.

The modeling methodology employed in this work makes use of cell-like and agent-like automata, which allows the descriptive and explorative process to be focused on property- and resident-specific hypotheses. It also allows for scaling of gentrification dynamics across micro- and macro-levels in an intuitive and seamless manner, with the advantage that the novelty of a complex adaptive systems approach can be employed.

The simulation results demonstrate that the model is adequate to capture theoretical dynamics of gentrification and is powerful enough to allow for hypothesis-testing and scenario-evaluation across a wide variety of considerations.

Although it is only verified on a theoretical level, our model demonstrates the potential of this type of analysis as a springboard to explore more sophisticated models of gentrification,

perhaps with a longer research agenda of supporting decision-making for policy-makers, urban planners, developers, and residents.

We regard the work as successful in terms of its immediate goals. There is room for improvement, however. One of the key issues is that it is necessary to gather micro-scale data before applying the model in the context of real urban dynamics, as well as for model validation and determination of appropriate simulation time and scale.

Further exploration into more of the mechanisms of gentrification, residential mobility, and property upgrading is another future research direction. In our approach, the phenomenon of gentrification is understood in terms of demand- and supply-side theories, and this concept is implemented in simulation scenarios by simply introducing gentrifiers and gentrifiable properties. However, gentrification in the real world is much more complex in its urban dynamics. For example, top-down concepts such as issues of urban planning and political zoning should be taken into consideration, while this study focused on bottom-up approaches. Other, less tangible, factors are likely important but are incredibly difficult to model: social biases, cultural factors, etc. In terms of the mechanism of residential mobility, this work used a utility function, which is derived from the idea of a hedonic approach, and residential mobility is determined by six variables. It is important to empirically determine the significant variables for housing choice behaviors. Which variables are critical and relevant components for housing choice behavior in the utility function, for example? This could be settled empirically by statistical methods ahead of model parameterization, but this is difficult in the absence of real micro-scale survey data. Nevertheless, we regard our model as a step in the right direction, ahead of examining these issues in future work.

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