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# Death and taxes: their implications for endogenous growth

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**Abstract**

Blanchard's explanation of consumption dynamics, when combined with a technology that does not exhibit diminishing returns, delivers a model consistent with many of the regularities in cross-country growth regressions. Longer life expectancies generate faster economic growth by affecting households' willingness to smooth intertemporally. Greater government expenditure or taxation lowers growth, but the effect of higher government debt is ambiguous. 

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## 1. Introduction

Blanchard (1985) has provided an influential model that sets a model of overlapping generations in continuous time. Chief among the contributions is the derivation of an explanation of consumption that depends on real wealth – which can include government bonds. This letter shows how this can coexist with steady-state growth when technology exhibits constant or increasing returns to scale in a factor that can accumulate. The richness of the Blanchard model when set in a framework of endogenous growth allows a discussion of the effects of life expectancy, tax policy, and government spending on growth prospects. This simple model has a number of testable implications, including several already captured in work on cross-country regressions of the sort documented by Barro (1997).

## 2. The model

Blanchard (1985) derives the optimal path of consumption of households who maximize utility that is additively time separable with unit risk aversion when confronted with the random chance of death.

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Lifetime welfare for a representative household (or a single cohort in the continuum of generations) can be represented as:

$$\int_t^{\infty} \ln(c_s)\, e^{(\theta+p)(t-s)} ds \tag{1}$$

where $\theta$ is the subjective rate of time discount and $p$ is the probability of death, which implies that average life expectancy is $1/p$. Blanchard shows under a consistent aggregation scheme across households that the flow of aggregate consumption per unit time, $C$, is given by:

$$\dot{C} = (r - \theta)\, C - p(p + \theta)W \tag{2}$$

where $r$ is the real instantaneous risk-free interest rate and $W$ is real wealth.[1] Because households may not live to see the future taxes levied to meet the interest payments on the debt, both government bonds, $B$, and capital, $K$, are part of net wealth, $W$.

The part of production of the single type of good that is not consumed is converted into fixed capital, which accumulates according to:

$$\dot{K} = Y - C - G - \delta K \tag{3}$$

net of depreciation at a constant rate, $\delta$. Government spending, $G$, is assumed to be perfectly wasteful – it does not enter households utility – and taxes, $T$, are of the lump-sum variety. The government must issue debt to fill any overage of spending and interest service over tax receipts, as in:

$$\dot{B} = rB + G - T \tag{4}$$

Modern models that make the growth rate of output endogenous rely on some mechanism that allows constant, or even increasing, returns in a factor that can accumulate. This letter adopts the simplest mechanism, presented by Barro (1990), among others, that output is produced according to a technology that is linear in capital:

$$Y = AK \tag{5}$$

Among the simplifications implied by Eq. (5) is that the real instantaneous return to capital is constant,

$$r = A - \delta \tag{6}$$

which is also the rate of return that government bonds must provide

[1] The dot denotes the derivative with respect to time and time subscripts will be suppressed where possible.

## 3. Life expectancy and growth

To highlight the role of life expectancy in influencing steady-state growth, first assume away any role for the government – that is, $G$, $T$, and $B$ are all identically zero. In that case, two equations determine the evolution of the growth rates of consumption and the capital stock,

$$\frac{\dot{C}}{C} = (A - \delta - \theta) - p(p + \theta)\frac{K}{C} \tag{7}$$

$$\frac{\dot{K}}{K} = A - \frac{C}{K} - \delta \tag{8}$$

If the economy were to attain steady-state growth, both consumption and the capital stock must expand at the same rate, say $\sigma$. In that case, Eqs. (8) and (9) are two relationships in two unknowns – the initial consumption–capital ratio, denoted $c$, and the steady-state growth rate, $\sigma$.

Those two equations are most intuitively solved graphically, as in Fig. 1. The consumption–smoothing relationship provides a positive, nonlinear relationship between long-run growth and the consumption–capital ratio, because a high level of $c$ implies that wealth is low relative to consumption, spurring expected consumption growth. The capital accumulation equation, in contrast, relates $c$ and $\sigma$ negatively along a line with a slope of unity (in absolute value), reflecting the national income identity that as more output is consumed, less is available to be added to the capital stock. There will be an interior solution to that problem as long as

$$(A - \delta)(A - \delta - \theta) > p(p + \theta) \tag{9}$$

And the household planning problem will be well defined in that solution provided that

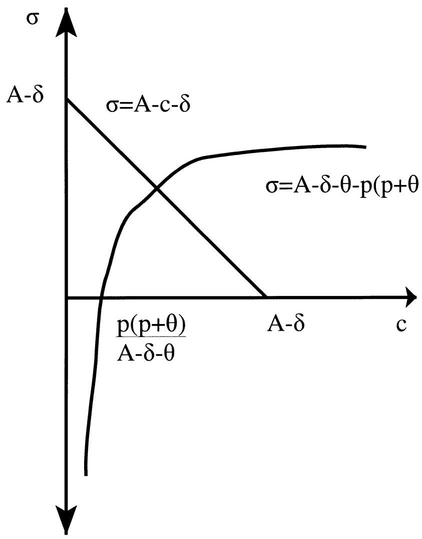


Fig. 1. Determination of steady-state growth.

$$\sigma < \theta + p \tag{10}$$

If these two conditions are met, then there is a unique choice of an initial consumption–capital ratio that is feasible given current production and that subsequently generates constant economic growth at the rate $\sigma$.[2]

Notice that households' sense of mortality – captured in the probability of death and subjective rate of time preference – influences only the consumption–smoothing relationship and not the national income identity. Hence, if average life expectancy were to shorten – $p$ increase – the consumption–smoothing condition in Fig. 1 would shift rightward, resulting in a higher consumption–capital ratio and a lower rate of steady-state growth in output. Households that expect a shorter life consume more and save less today to the detriment of long-run economic growth.

This relationship is consistent with the strong empirical result emerging from panel regressions on the determinants of economic growth. As explained in Barro (1997, p. 19), life expectancy (or highly correlated variables such as infant mortality or other indicators of health status) is positively related to income growth. That association is usually justified as holding because longer life expectancy raises human capital, allowing more output to be produced. This model offers a different link: Longer-lived households are more patient, making them willing to substitute more across time and, therefore, generating more saving and investment. In that regard, this matches the result of Ogaki et al. (1996), who estimated consumption–smoothing relationships across regions of developing countries. Among their findings was that estimated rates of time discount align negatively with rates of regional income growth.

## 4. The government and growth

We can introduce a role for the government and not jeopardize steady-state income growth only if government policies are designed to expand at the same, endogenous rate as income. That is, we require that

$$T_t = T_0\, e^{\sigma t} \tag{11}$$

$$G_t = G_0\, e^{\sigma t} \tag{12}$$

thereby limiting the choice to the initial levels of lump-sum taxation and government spending, with the obvious potential for feedback on $\sigma$. Even there, those initial choices cannot be independent as they must satisfy the flow budget restraint. To simplify matters, we can either take initial government spending as exogenous and allow taxes to adapt or initial taxes as exogenous and allow spending to adapt.

In either case, the model simplifies to three equations, representing consumption smoothing, the national income identity, and the government's flow budget restraint. Along a balanced growth path,

[2] Intuitively, the first condition guarantees that the consumption–smoothing relationship cuts the national income identity in Fig. 1 in the positive orthant, while the second requires that future consumption grows more slowly than the rate at which it is discounted to keep expected lifetime utility bounded.

consumption, the capital stock, and net government debt must all expand at the same rate, $\sigma$, and can be described in terms of ratios to the capital stock, as in:

$$\sigma = (A - \delta - \theta) - p(p + \theta)\frac{1+b}{c} \tag{13}$$

$$\sigma = A - c - g - \delta \tag{14}$$

$$\sigma = A + \frac{g}{b} - \frac{t}{b} \tag{15}$$

where the lower case quantity variables, $c$, $b$, $g$, and $t$ are all measured as ratios to the capital stock.

### 4.1. Exogenous government spending

If the starting level of government spending is exogenous, then Eqs. (13) and (14) determine the initial consumption–capital choice and subsequent rate of economic growth. Given that outcome for growth, the government must set taxes by Eq. (15) to ensure that its stock of bonds grows in line with income. The determination of $c$ and $\sigma$ given $g$ can be solved graphically, subject to similar feasibility and boundedness conditions, as in the previous section. Relative to an initial interior solution, an increase in government spending pulls the linear national income identity inward, lowering both the consumption–capital ratio and steady-state growth. Not surprisingly, higher government spending crowds out consumption and saving, which lowers investment and long-run income growth (as is familiar from Barro, 1990). New to this model, though, is the result that a higher initial stock of bonds shifts the consumption–smoothing condition rightward and is associated with a higher consumption–capital ratio and lower steady-state growth. Simply, because government bonds are a part of net wealth, a higher inherited level of $b$ supports higher consumption at the expense of saving and investment.

### 4.2. Exogenous taxation

If the starting level of taxation is exogenous, then government spending must be adjusted by Eq. (15) so as to produce a flow of debt that matches the expansion of income. Substituting that relationship explaining government spending into the national income identity (Eq. (14)) yields:

$$\sigma = A - \frac{c + t + \delta}{1 + b} \tag{16}$$

which, when combined with Eq. (14), determines the initial consumption–capital ratio and subsequent economic growth. Again, that solution can be arrived at through simple and obvious modifications to the graphical apparatus in Fig. 1. Relative to an initial equilibrium, an increase in taxes, because it makes possible higher government spending, shifts the national income locus inward, lowering the consumption–capital ratio and steady-state growth. The effect of higher debt burdens, however, is ambiguous. As in the prior case, increased government debt, because it raises wealth, induces households to spend more, working to lower saving and investment. At the same time, higher government interest payments at the same level of taxes lowers the extent to which the government

Table 1
Steady-state growth rates ($\sigma$) consistent with alternative life expectancies and government policies percent[a]

| Average life expectancy ($l/p$) (years) | Exogenous government spending ($G/Y$) | | Exogenous taxes ($T/Y$) | |
|---|---|---|---|---|
| | 10 percent | 20 percent | 20 percent | 30 percent |
| 40 | 1.93 | 1.40 | 1.76 | 1.21 |
| 50 | 2.49 | 1.99 | 2.32 | 1.79 |
| 60 | 2.88 | 2.40 | 2.71 | 2.20 |
| 70 | 3.15 | 2.70 | 2.99 | 2.50 |
| 80 | 3.36 | 2.93 | 3.21 | 2.73 |

[a] Note: The cell entries represent solutions to the model given by Eqs. (13)–(15) assuming that: $A=0.15$, $\theta=0.05$, $\delta=0.05$, and $B/Y=0.5$.

can spend, fostering investment. In that regard, a higher level of debt is a constraint on government spending.

## 5. Conclusion

Blanchard's (1985) explanation of the dynamics of consumption, when combined with a specification of technology that does not exhibit diminishing returns, delivers a model general enough to be consistent with many of the empirical regularities in cross-country growth regressions. The main results can be summarized by simulations of the model, given in Table 1, for fairly representative behavioral parameters and initial conditions listed at the bottom of the table. Across the rows of the table, the parameter $p$ is varied to produce average life expectancies ranging from 40 to 80 years. The first two columns give the steady growth rate consistent with fixed government spending equal to 10 and 20 percent of income, respectively, and allow taxes to vary endogenously.[3] Columns 3 and 4 repeat the exercise assuming taxes are set exogenously at 20 and 30 percent of income, respectively, and government spending is determined endogenously. The main point of this letter can be seen by scanning the rows: Longer lives are associated with higher growth rates. Moreover, the effect is nonlinear, in that progress in extending lifespans raises growth the most when life expectancy is low. Also note that higher government spending (column 2 versus column 1) or higher taxes, which make higher government spending possible, (column 4 versus column 3) lowers growth for any life expectancy.

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[3] It is more intuitive to deal with shares of income, but the model uses the capital stock to scale the endogenous variables. Given the specification of technology – the parameter $A$ – the translation is simple.

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