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ABSTRACT

Economic Linkages Across Space*

We develop a diagrammatic framework that can be used to study the economic linkages between regions or cities. Hitherto, such linkages have not been the primary focus of either the theoretical or empirical literatures. We use the framework to analyse the impact of shocks that occur in one region (eg productivity improvements or increases in housing supply) on other regions, highlighting the key adjustment mechanisms and their long run implications for incomes, the cost of living, and the spatial distribution of population. Our general approach provides a framework encompassing both the New Economic Geography and Urban Systems literatures. We link our approach to these literatures and review empirical studies that quantify the key mechanisms that we have identified.

JEL Classification: R00 and R58
Keywords: new economic geography, spatial linkages, urban and regional policy and urban systems

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1. Introduction.

While the primary impact of an economic change may be focused in a particular city or region, its effects may also be felt in other regions. There are interdependencies across places so that what happens in one region has implications not only for this location but for other regions as well. The objectives of this paper are to investigate these linkages, identifying the channels through which these effects operate and their net impact. The issue is important because, as we will see, under some circumstances changes that benefit one region also have positive effects on other areas. When this complementary relationship between regions holds, the impact of a shock is in some sense shared between regions. Under other circumstances, however, regions are in a competitive rather than a complementary relationship with one another. A positive shock to one region has a negative impact elsewhere, with the result that the effects on the first region are amplified at the expense of other regions.

Understanding the circumstances under which these alternatives occur is crucial for understanding regional inequalities, and for evaluating the effects of policy. Consider two examples. A policy measure raises productivity and hence wages in one city or region. Does this increase wages in other regions also, or are spatial differences in wages amplified by induced migration and consequent changes in earnings? An expansion in housing stock in a high price region is intended to narrow regional house price differentials. Does it do this, or is it offset – or even overturned – by induced migration and higher earnings in the growing region? The answers to these questions turn on whether regions are in a complementary or competing relationship with each other, terms that we will make precise in the course of the paper.

Our technique for investigating these issues and establishing the relationship between regions is primarily diagrammatic. We develop a framework that shows how two regions in an economy interact, and illustrates how there can be equilibrium spatial disparities. We use the framework to analyse the effect of a change in one region on another and to show how – depending on a few key elasticities – equilibrium adjustment may dampen or amplify shocks.

The framework is based on three key relationships. The first is the 'earnings-employment' relationship, which captures the supply side of the economy. The form of this relationship depends critically on whether there are increasing or decreasing returns to expanding employment in a region. The second is the 'cost-of-living' relationship, which captures the effects of levels of employment on prices of goods and assets in the region. It depends on characteristics of goods markets and, above all, land and housing markets. The third is the 'migration relationship', linking population movements between regions.

We express each of these relationships in a 'reduced form' way, and show how the interaction between regions depends on the shape of the relationships. Our focus is on the long-run equilibrium of the economy and, for clarity, we abstract from many of the real life frictions that may arise from imperfect information or adjustment costs. This approach has, we hope, the benefits of being relatively accessible and independent of the details of particular modelling approaches. However, these reduced form relationships summarise the micro-economic detail of a wide range of economic models, and it is important that they are linked to models in the literature. The later sections of the paper relate these relationships to both the theoretical and empirical literatures in economic geography, regional economics, and urban systems.

2. Linkages: a Diagrammatic Approach.

The main objective of the paper is to develop an analytical framework that can be used to understand the equilibrium of a multi-region economy and establish the comparative static effects of 'shocks' to one of the regions. The text develops the argument diagrammatically for a two region economy, while a more general algebraic analysis for a multi-region economy is presented in the appendix. We shall refer to the two regions, as N and S. The total labour force in the economy is fixed, and normalised at unity, but it can be divided between the two regions with S having share λ and N having $1 - \lambda$. We will assume that the labour force in each region is proportional to both population and employment, thereby abstracting from differences in demographic structure, in regional participation or employment rates, or commuting between regions.[1]

We focus on three key 'reduced form' relationships which can be captured in a four quadrant diagram.

[1] Exogenous differences can be captured by region specific parameters. Some sources of endogenous variation can be captured in our reduced form relationships (for example, participation depends on the wage which in turn depends on λ) but for clarity we leave these forces in the background.

Earnings-employment (EE): The first relationship we refer to as the earnings-employment relationship and it determines the wage that is paid to workers in a region as a function of the labour force in the region. Denoting wage rates in *N* and *S* by w_N and w_S, the earnings-employment relationship can be written as:

$$w_S = a_S w(\lambda), \qquad w_N = a_N w(1-\lambda) \tag{1}$$

where a_N and a_S are productivity parameters, and the function *w*(.) gives the relationship between earnings and employment. We will denote the elasticity of this relationship by η_w, noting that the elasticity may vary with λ. The shape of this relationship depends on technology and market interactions. For example, if product and labour markets are perfectly competitive and there are diminishing returns to labour then the function gives the marginal value product of labour, and $\eta_w \leq 0$.[2] If there are increasing returns (internal or external to firms) then it is possible that $\eta_w > 0$, and we discuss the forces that shape the relationship in this case in section 5. The relationship also depends on the mobility of other factors of production. In the within-country context we think of capital markets as being integrated, so the rate of return is the same in all regions and any diminishing returns would come from regionally immobile factors of production.

The earnings-employment relationship is illustrated in the top left quadrant of figure 1 by curve *EE*. Because we are interested in the linkages between regions the figure is constructed in terms of the division of the labour force between regions, and the relative values of the price variables in each region. Thus, the horizontal axis of the top-left quadrant is λ, the share of the labour force in *S*, and the vertical is relative wages, w_S/w_N. That is:

EE: $$w_S / w_N = (a_S / a_N)(w(\lambda)/w(1-\lambda)) \equiv W(\lambda), \tag{2}$$

where the function *W*(λ) summarises the relationship and is assumed to be invertible. The interpretation of the relationship is straightforward. Given a division of the labour force between the two regions, λ, the curve shows the associated relative wage w_S/w_N. The case illustrated is one with diminishing marginal returns to labour, $\eta_w < 0$, so that

[2] If the production function is Cobb-Douglas and other factors are fixed then $\eta_w + 1$ is equal to the elasticity of output with respect to labour.

an expansion of relative employment in S leads to a fall in the relative wage in S. We will look at the case where $\eta_w > 0$ in section 3.

Cost of living (HH): High relative earnings in a region should encourage migration into that region, but migration in turn has implications for the cost of living. There are several mechanisms through which population can affect the cost of living, the most direct of which is through the fact that people consume space. Increasing the labour force in a region bids up the prices of houses (and residential land) and so raises the cost of living. This relationship is steeper the less elastic is housing supply (eg due to planning regulations) and the greater is the share of housing in expenditure.

We denote the cost of living in each region by h_S , h_N and assume, for the moment, that it is a function only of the labour force in the region. Thus,

$$h_S = h(\lambda / b_S), \qquad h_N = h((1-\lambda) / b_N) \qquad (3)$$

where, once again, b_S and b_N are shift parameters and $h(.)$ is the functional relationship linking the cost of living to the labour force. The elasticity of the cost of living with respect to the labour force is denoted η_h , and may vary with λ. The parameters, b_S and b_N , can be interpreted as exogenous factors that shift the supply of (or demand for) land, so an equiproportional increase in b_S and λ has no effect on the cost of living in S. Taking the ratio of these expressions,

$$\textbf{\textit{HH:}} \qquad h_S / h_N = h(\lambda / b_S) / h((1-\lambda) / b_N) \equiv H(\lambda) \qquad (4)$$

where the function $H(\lambda)$ summarises the relationship, and is assumed to be invertible. This relationship is illustrated in the bottom right quadrant of figure 1, drawn with $\eta_h > 0$, so that greater population is associated with higher cost of living.

While we have written the cost of living as a function of the labour force, λ, notice that the expression can be interpreted more generally. For example, the demand for housing may depend also on wages. Indeed, as we discuss in section 5, the income effect on prices may even be bigger than the population effect. However, as wages are themselves a function of λ (equation (1)), the function $h(.)$ is a reduced form that can incorporate these effects. The implications for the shape of $h(.)$ will depend on the nature of the function $w(.)$ as discussed in the appendix.[3] Other factors may also enter this relationship, including goods prices and intra-regional commuting costs, and these are, again, discussed further in section 5.

Migration (MM): We assume that workers are potentially mobile between regions and that they move in response to *real* wage differences between regions. Long run equilibrium occurs therefore when real wages, adjusted by amenity values, are equalised across regions. Since real wages are nominal wages divided by the cost of living index, the migration equilibrium schedule is determined by the condition,

$$\textbf{\textit{MM}}: \quad c_S w_S / h_S = c_N w_N / h_N \text{ or equivalently } w_S / w_N = (h_S / h_N)(c_N / c_S), \qquad (5)$$

where c_S and c_N are shift parameters that reflect, for example, amenities in the two regions. If $c_S = c_N$ then this migration equilibrium schedule is simply the 45% line as illustrated by the line *MM* in the top right hand quadrant of figure 1. Above the line workers in *S* are better off than those in *N*, so there is a tendency for migration from *N* to *S*, and conversely below the line.

With three economic relationships for three pairs of endogenous variables (the labour force in each region, wages and costs of living) the fourth quadrant is conceptually redundant, and the line *LL* is simply the 45^{o} line, matching values of λ across quadrants.

3. Equilibrium.

Together, these relationships determine the long run equilibrium distribution of workers between regions and the associated levels of earnings and costs of living. The full long run equilibrium of this system occurs when values of λ, w_S / w_N, and h_S / h_N are consistent with all relationships holding simultaneously. This long run equilibrium could take time to achieve if people do not respond instantly to real wage differentials. If adjustment takes time, expectations may matter for both the nature of long run equilibrium and for the adjustment path between equilibria (e.g. whether there is overshooting). These issues, while beyond the scope of this paper, are certainly

[3] Comparative static analysis of a change in productivity parameters would then shift this relationship, an effect that we ignore in section 4 but discuss in the appendix.

interesting and the subject of a small but growing literature. Henceforth, we abstract from them and consider only the long run equilibrium as defined here.[4]

In the simplest case, in which the two regions are symmetric ($a_S / a_N = b_S / b_N = c_S / c_N = 1$) there is an equilibrium at which $\lambda = ½$ and relative values of all price variables are unity. Before applying the framework we need to spend some time outlining how equilibrium is attained, and distinguishing between three different cases – distinctions that will be crucial when we come to comparative static analysis.

3.1 Complementary regions:

Figure 1 illustrates the case of 'complementary regions'. The equilibrium is shown by the points indicated by circles and joined by the dashed lines.

To understand the diagram it is helpful to consider the following thought experiment. Suppose that the initial situation is one in which employment in *S* is relatively high, as at point *A* on the *LL* curve. Tracing up to the *EE* curve gives the value of relative wages corresponding to relative employment at *A*. Similarly, tracing over to the *HH* curve gives the relative cost of living. Looking at the *MM* quadrant, we see that these levels of relative wages and living costs correspond to the point *A'* which is below the *MM* curve, meaning that real wages in *N* are above those in *S*. As we move point *A* along the *LL* line in the bottom left quadrant, relative wages and relative living costs change, tracing out the dashed curve *ZZ* in the top right quadrant. Thus the *ZZ* curve traces out combinations of relative wages and relative living costs, given the employment-earnings and cost of living relationships.

Now, recall that the *MM* curve gives the combinations of relative wages and relative living costs at which real incomes in the two regions are equal. Thus, the long run equilibrium must be at the intersection of *MM* and this derived curve *ZZ*. At this point relative wages and living costs are consistent with the division of the labour force between regions, and there is no incentive for workers to migrate from one region to the other.

Notice that this equilibrium is stable, in the sense that labour migration is an equilibrating force. To see this, once again consider starting at labour allocation *A*.

[4] An additional complication, which could be more easily incorporated, arises from the presence of a fixed cost of moving. In this case, the migration equilibrium schedule would be characterised by a band with the upper curve of that band identifying real wage differences that are sufficiently high to overcome the fixed cost of moving and lead to migration from north to south (and vice versa for the lower band). All points within this band would then represent possible long run equilibria but with the remaining analysis essentially unchanged.

Workers in *S* have a lower level of real income than workers in *N* (point *A'*). As a result, workers migrate from *S* to *N* moving the economy along the *LL* line in the direction of the arrow. As λ declines, w_S/w_N increases (the *EE* relationship) and h_S/h_N falls (the *HH* relationship), moving along *ZZ* in the direction of the arrow until the equilibrium is reached.

For reasons that will become clear when we turn to comparative statics, we define the configuration illustrated in figure 1 as the 'complementary case'. The aspect of the configuration that matters for comparative statics is that the *ZZ* curve is downwards sloping. The *ZZ* relationship is derived by eliminating λ from the *EE* and *HH* curves,

ZZ: $$w_S / w_N = W\left(H^{-1}\left(h_S / h_N\right)\right), \tag{6}$$

where $W\left(H^{-1}(.)\right)$ is the composition of the *W* function in equation (2) and the inverse of the *H* function in equation (4). If the functions *w*(.) and *h*(.) are isoelastic, so the *EE* and *HH* equations, (2) and (4), are respectively

$$w_S / w_N = \left(a_S / a_N\right)\left(\lambda/(1-\lambda)\right)^{\eta_w}, \qquad h_S / h_N = \left(b_S / b_N\right)^{-\eta_h}\left(\lambda/(1-\lambda)\right)^{\eta_h},$$

then the *ZZ* relationship takes the form,

ZZ: $$w_S / w_N = \left(a_S / a_N\right)\left(b_S / b_N\right)^{\eta_w}\left(h_S / h_N\right)^{\eta_w/\eta_h}. \tag{6'}$$

In our analysis we do not impose that these functions are globally isoelastic – some of the curvatures illustrated in following diagrams certainly violate this property. The isoelastic form is nevertheless useful as a way of capturing relationships in the *neighbourhood of the symmetric equilibrium.* At this point, elasticities have the same values in both regions, and we will describe the slopes of relationships at the symmetric equilibrium in terms of elasticities. Thus, from now on elasticities are always understood to be evaluated in the neighbourhood of the symmetric equilibrium. The configuration of figure 1 therefore holds because the *ZZ* schedule is downward sloping at the symmetric equilibrium, i.e. $\eta_w/\eta_h < 0$. Notice that, if $\eta_h > 0$, then this obviously requires a negative elasticity of earnings with respect to employment, $\eta_w < 0$.

3.2 Competing regions:

The case illustrated in figure 1 assumes that there are diminishing marginal returns to labour ($\eta_w < 0$), so that along the *EE* curve higher levels of employment are associated with lower wages. But at the heart of much of the literature on urban systems and new economic geography is the idea that this relationship may be positive as a consequence of increasing, rather than diminishing, returns to activity in an area. The mechanisms underlying this are discussed in more detail in section 5, but for now we simply consider the implications of this for the equilibrium. The *EE* curve is now positively-sloped as shown in figure 2, and so higher relative employment in *S* also implies higher relative wages. The change in the slope of the *EE* curve affects the slope of *ZZ* schedule and this has major implications for the comparative static properties of the model as we shall see in section 4. It is the positive slope of the *ZZ* schedule that defines this case as 'competing regions' and, in terms of elasticities this occurs when $\eta_w / \eta_h > 0$ (equation (6')); if $\eta_h > 0$, then this obviously requires a positive elasticity of earnings with respect to employment, $\eta_w > 0$.

Although the slope of the *ZZ* curve has reversed, the equilibrium illustrated in figure 2 is still stable. Relatively high values of λ map into points on *ZZ* below the *MM* line, meaning that real incomes in *S* are low relative to those in *N* and so there is migration from *S* to *N* which moves λ towards its equilibrium value. This stability comes about because the slope of *ZZ* is less than that of *MM*, and for this we require that the elasticities satisfy $\eta_h > \eta_w > 0$. That is, as the labour force in a region increases, the cost of living needs to rise proportionately faster than earnings. Again, for reasons that will become clear when we turn to comparative statics, we refer to this situation as the "competing case".

3.3 Divergent regions:

The symmetric equilibrium of this model is unstable if increasing returns in the earnings-employment relationship are sufficiently strong, relative to the increasing costs in the cost of living relationship. This case is illustrated in figure 3. The *EE* curve is as in figure 2 but the cost of living curve, *HH*, is drawn differently. For the moment, focus only on the area around the symmetric equilibrium and note that the cost of living now responds less sharply to population changes. This in turn produces a *ZZ* schedule which is steeper than the *MM* curve, and as a result the symmetric equilibrium is unstable. In this case, if λ is relatively high then *ZZ* lies above *MM*; real incomes in *S* are above those in *N* and migration from *N* to *S* increases λ further. This situation results when the underlying parameters are such that $\eta_w / \eta_h > 1$.

If the equilibrium is unstable, what happens? There may be no other intersection of the *MM* and *ZZ* curves in which case one of the regions empties out, with λ going to zero or one. Alternatively, it may be the case that as population levels in the two regions become very unequal, the cost-of-living becomes more responsive to further changes in population, resulting in a 'bendy shaped' *HH* curve, as shown in figure 3.[5] This shape transfers to the *ZZ* curve, and implies that there are two asymmetric equilibria, in addition to the symmetric equilibrium already discussed. These are both stable (*ZZ* is less steep than *MM* at the point of intersection) and 'competing' (*ZZ* has positive gradient, as in figure 2). Thus, regions with similar underlying characteristics may be observed to have quite different population levels, wages and living costs.

3.4 Regime summary:

Figure 4 summarises the relationship between the slopes of the relationships (measured by elasticities at the symmetric equilibrium) and the complementary, competing and divergent cases we have outlined. If $\eta_h > 0$, as assumed in all the figures in the paper, then increasing η_w moves the system from complementary to competing to divergent, as illustrated. Increasing returns, $\eta_w > 0$, is necessary for regions to be competing and if these are strong enough they overturn the force for dispersion captured by $\eta_h > 0$, giving divergence. If $\eta_h < 0$ then this force for dispersion is removed, and regimes are as illustrated on the left hand side of figure 4. The models underlying this case – in particular Krugman's (1991a) core-periphery model – are discussed in section 5.

[5] This requires that the elasticity of the *HH* curve varies. This would happen if, for example, congestion rises faster with population for small cities (perhaps because it is inefficient to offer public transport) and large cities than for medium size cities. Varying elasticity on the *EE* curve could also achieve this if there were diminishing returns over some range of city sizes and increasing returns over others.

4. Regional Linkages.

We now turn to investigating the comparative static properties of the model. To do this we assume that one region (*S*) is affected by some exogenous 'shock', and then see how inter-regional economic adjustment restores the equilibrium of the economy. For obvious reasons we only look at changes to a stable equilibrium, and we commence with a shock that shifts the *EE* relationship, i.e. a productivity shock.

4.1 Productivity shock:

Region *S* experiences a positive productivity shock, such as an infrastructure investment or some other supply side improvement. The direct effect of the shock is to shift the *EE* schedule upwards, as illustrated by the vertical arrows in figure 5a (for the case where the regions are complementary) and figure 5b (competing regions). This shows that at given employment levels, the earnings of labour in *S* are increased. This upwards shift in *EE* shifts the *ZZ* schedule upwards by an identical amount[6], and the new (long-run) equilibrium is at the intersection of this new *ZZ* schedule and the *MM* line (illustrated by the outer rectangle of dashed lines in figures 5a and 5b) Qualitatively, the effects of the positive shock to productivity are as expected. There is an increase in wages in both regions and an increase in relative wages in *S*, w_S/w_N. There is migration to *S*, and hence an increase λ, population and employment in *S*. This in turn leads to an increase in the relative cost of living in *S*, h_S/h_N, offsetting the change in relative wages.

How does the shape of each of the relationships *EE* and *HH* affect the nature of the linkages between areas? For the complementary case illustrated in figure 5a, the relative wage change is smaller than the initial productivity shock, as can be seen by comparing the magnitude of the shifts in *EE* and *ZZ* with the associated wage change. This is the complementary relationship: a beneficial shock in *S* first increases wages in *S*, but then interregional migration draws labour into *S* from *N*, and as it does so wages in *S* fall and wages in *N* rise, partially offsetting the initial effect of the shock. At the same time, these population movements produce changes in relative costs of living with house prices in *S* increasing while those in *N* decline, offsetting the change in relative wages. Overall, the beneficial shock in *S* produces higher real wages in both regions,

[6] This assumes that the productivity shock does not have any direct effect on the cost of living. This assumption is relaxed in the appendix.

with higher employment and house prices in *S,* but an absolute fall in employment and house prices in *N*. The magnitude of the changes depends on the elasticities of the relationships. For example, if housing supply is very price inelastic then the *HH* curve is flat, and small changes in relative population cause large changes in relative living costs. In this case, the shock is associated with small changes in relative populations, λ, and relatively large changes (and consequent differences) in both wages and living costs. Conversely, very price elastic house supply means that the shock causes large movements in population and small long run differences in costs of living and wages. In the limit with *HH* and hence *ZZ* vertical, relative costs of living are unchanged and population movement brings wages back to equality.

How are things different in the case of competing regions? It is helpful to think first about the borderline case, in which the *EE* schedule is horizontal. Relative wages do not depend on employment levels, so the change in relative wages must be equal to the productivity shock. All adjustment occurs through population movement and its consequent effects on living costs. When the *EE* relationship is positively-sloped then we move from the complementary case of figure 5a to the competing region case of figure 5b. The productivity shock draws labour to *S* from *N* as before, but now this migration causes further increases in earnings in *S*, while decreasing earnings in *N*. The price changes are therefore relatively large, and in particular the wage changes are amplified. Wages in *S* rise by *more than* the initial productivity shock, while wages in *N decline* as labour emigrates. Correspondingly large changes in relative population and in the cost of living are necessary therefore in order to equate real wages and restore equilibrium.

Algebraic expressions for all these effects are given in the appendix, and are summarised in Table 1. The productivity shock is of magnitude $\hat{a}_S$ (^ denotes a proportional change). If $\eta_w < 0$ (complementary regions, figure 1 and 5a) then there is a less than proportional increase in wages in *S* (i.e. $\hat{a}_S > \hat{w}_S$) and wages in *N* rise. But if $\eta_w > 0$ (competing regions, figures 2 and 5b) then migration amplifies (rather than moderates) the effect in *S*, giving a larger than proportional wage increase in *S* and wage reduction in *N*, $\hat{w}_S > \hat{a}_S > 0 > \hat{w}_N$. In both cases the cost of living (and house prices) in *S* increase while they fall in *N*, the magnitude of these effects being greater the larger is η_w. The change in the relative living costs in the two regions, $\hat{h}_S - \hat{h}_N$, is less than the productivity shock if regions are complementary, but greater than the productivity shock if regions are competing.

Table 1: Equilibrium responses to an increase in productivity in south

$\hat{a}_S > 0$, implying population movement $\hat{\lambda} > 0$.

	Proportionate change in wages: $\hat{w}_S, \hat{w}_N$.	Proportionate change in cost of living: $\hat{h}_S, \hat{h}_N$.
Complementary: $\eta_h > 0 > \eta_w$	$\hat{a}_S > \hat{w}_S > \hat{w}_N > 0$	$\hat{a}_S > \hat{h}_S - \hat{h}_N > 0$ $\hat{h}_S > 0, \quad \hat{h}_N < 0$
Constant returns: $\eta_h > 0 = \eta_w$	$\hat{a}_S = \hat{w}_S > \hat{w}_N = 0$	$\hat{a}_S = \hat{h}_S - \hat{h}_N > 0$ $\hat{h}_S > 0, \quad \hat{h}_N < 0$
Competing: $\eta_h > \eta_w > 0$	$\hat{w}_S > \hat{a}_S > 0 > \hat{w}_N$	$\hat{h}_S - \hat{h}_N > \hat{a}_S > 0$ $\hat{h}_S > 0, \quad \hat{h}_N < 0$

Stability requires $\eta_h > \eta_w$

4.2 Land supply shock:

As a second experiment, we consider the effect of a shock that enables *S* to have a higher population with unchanged cost of living. Perhaps the best example of this is a change in land use regulations that allows *S* to expand the stock of land available for housing. The effect is to shift the *HH* curve left or downwards (a lower cost of living for the same size labour force), this translating into a similar size shift to the left of the *ZZ* curve.

The effects for complementary regions are illustrated in figure 6a, and the new equilibrium is at the points on the dashed rectangle lying to the "south-west" of the old equilibrium. The effects can be traced out as follows. With given population, the shock leads to a fall in house prices in *S*, reducing living costs and raising real wages. This leads to migration to *S* from *N* and, with diminishing returns to labour, wages fall in *S* and increase in *N*. The net result (once real wages are equalised by migration) is lower house prices and lower cost of living in both regions, with the house price decline greater in *S* than in *N*, to compensate for the lower wages in *S* (row 1, table 2).

If regions are competing then results change radically. With an upwards sloping *EE* curve (figure 6b) the shift establishes the new equilibrium on the *outer* rectangle of dashed lines. As before, the initial fall in house prices in *S* leads to in-migration, but this now raises earnings in *S* and reduces them in *N*, widening the gap in real wages. This attracts further inflows of labour, and the process is equilibrated only when the relative cost of living in *S* has increased sufficiently, as shown in the figure. Increasing the stock of housing in *S* must therefore increase the relative cost of living in *S*, h_S/h_N. If the *EE* schedule is sufficiently steep, the new equilibrium requires that absolute house prices in *S increase* in response to the construction of more houses! The intuition here is that of agglomeration. Increasing returns are large enough that population continues to flow into the region until choked off by higher house prices. Formally, this case arises if $\eta_w > \eta_h / K$, where *K* is the number of regions (so $K = 2$ in the diagrammatic analysis). This is a condition that is quite consistent with stability of equilibrium, and the full listing of possibilities is given in table 2.

Table 2: Equilibrium responses to an increase in land supply in south.

$\hat{b}_S > 0$, implying population movement $\hat{\lambda} > 0$.

	Proportionate change in wages: $\hat{w}_S, \hat{w}_N$.	Proportionate change in cost of living: $\hat{h}_S, \hat{h}_N$.
Complementary: $\eta_h > 0 > \eta_w$	$\hat{w}_S < 0, \hat{w}_N > 0$	$0 > \hat{h}_N > \hat{h}_S$
Constant returns: $\eta_h > 0 = \eta_w$	$\hat{w}_S = \hat{w}_N = 0$	$0 > \hat{h}_N = \hat{h}_S$
Competing: $\eta_h > \eta_w > 0$	$\hat{w}_S > 0,\ \hat{w}_N < 0$	$0 > \hat{h}_S > \hat{h}_N$
Competing: $\eta_h > \eta_w > \eta_h / 2 > 0$	$\hat{w}_S > 0,\ \hat{w}_N < 0$	$\hat{h}_S > 0 > \hat{h}_N$

Stability requires $\eta_h > \eta_w$

These two examples illustrate the importance of understanding whether regions are complementary or competing, and show how qualitative, as well as quantitative effects of policy change depend on the elasticities of some key relationships. Similar analyses can be conducted for other parameter changes – such as amenity changes – or for combined packages of policy change. Rather than undertaking more of these exercises, we now turn to the mechanisms underlying our reduced form relationships.

5. The Earnings – Employment Relationship

The analysis presented above is based on reduced form relationships. In the remainder of the paper we go behind these relationships, outlining the mechanisms that underpin them and linking these to the existing literature, theoretical and empirical. We start, in this section, with the *EE* schedule which, as we have seen, is crucial in determining whether regions are complementary, competing or divergent.

A standard perfectly competitive model of production implies that additional employment in a region either reduces earnings or leaves them unchanged, $\eta_w \leq 0$. This is because, with non-increasing returns to scale, perfect competition, and some regionally fixed factor (call it capital), adding labour to a region increases the labour-capital ratio, thereby reducing the marginal product of labour. If capital were mobile or additional labour could be employed by changing the sectoral structure of production while holding labour-capital ratios in each sector constant (as in Heckscher-Ohlin trade theory) then earnings would be independent of employment, $\eta_w = 0$.[7]

Once we move outside the standard competitive framework it is possible that increased employment may be associated with higher wages, and the literature focuses on three basic mechanisms -- a classification that fits with that of Marshall (1890). The first is a market failure in the product market, modelled in the 'new economic geography' literature as an interaction between firm-level returns to scale and transport costs (section 5.1). The second is market failure in input markets, particularly the labour market (section 5.2), and the third is a range of technological externalities (section 5.3).

5.1 New economic geography:

At the heart of 'new economic geography' models is a trade-off between two features. First, production within firms exhibits increasing returns to scale, implying that each firm wants to concentrate its production in one location, not spread it between many. Second, there are trade costs in shipping goods, implying that firms gain from producing close to their markets. The combined effect of these two features is that firms seek to locate in regions with good market access, and this will tend to bid up wages in these regions. But good market access is itself a consequence of having a

[7] Factor prices are invariant with respect to a region's endowment, see Leamer and Levinsohn (1995)

large population. This provides the basis for a positive relationship between population and wages – i.e. an upward-sloping *EE* schedule.

The formal modeling of this follows from Krugman (1991a). The manufacturing sector contains firms that have increasing returns to scale and produce differentiated products, as in Dixit and Stiglitz (1977). Product differentiation means that firms engage in intra-industry trade, with each firm supplying all regions, although the presence of transport costs means that firms' sales are skewed towards their 'home' market. The industry is monopolistically competitive, so the number of firms in each location is determined by a zero profit condition. [8] This condition is the key part of the *EE* relationship, and in the literature is sometimes referred to as the wage equation (Fujita, Krugman and Venables 1999). It implicitly defines the wage at which firms in each region make zero profits, and this wage is an increasing function of regional expenditure or market size. Adding an income/ expenditure relationship which makes expenditure in each region an increasing function of employment, gives a positive relationship between employment and earnings, $\eta_w > 0$ and makes regions competing rather than complementary.

Krugman's model is a fully specified general equilibrium model, from which it is possible to derive exact conditions under which the relationship between regions switches from being competing to being divergent, with clustering of manufacturing activity in one region. This turns out to be more likely the greater is the proportion of economic activity that is mobile (e.g. non-agricultural activity) and the lower are transport costs. One of the main points that Krugman sought to make was that economic integration – lowering transport costs – could trigger regional divergence.

The Krugman model has been developed in many directions, and we note just two of them. One is that the results are not dependent on the standard Dixit-Stiglitz model of product differentiation and monopolistic competition. The fact that a large market is attractive to firms, so leading to higher equilibrium wages in the larger market, holds for other forms of competitive interaction between firms (Irmen and Thisse, 1998, Coombes and Lafourcade 2003).

Another extension was to add intermediate goods (Venables 1996, Krugman and Venables 1995). In the basic Krugman model proximity to a large market tends to raise profits (or, with free entry, wages). The presence of intermediate goods means that

[8] The fundamental market failure in the model is increasing returns at the level of the firm, meaning that firms do not divide their production between all regions.

market size depends not just on final consumers, but also on the presence of other firms who demand intermediate goods, and the effect of this is to amplify the value of being in an economically large region. Furthermore, firms also save transport costs from locating close to their suppliers. These effects (sometimes referred to as forward and backward linkages) serve to create increasing returns within and between manufacturing sectors, so much so that agglomeration of manufacturing can occur even if labour is immobile. The main point for current purposes is that the presence of intermediate goods amplifies the positive relationship between earnings and employment, so will tend to make η_w larger.

5.2 Thick market effects:

Large markets are often more efficient than small markets – an argument that applies in labour and capital markets as well as in product markets. In equilibrium, these 'thick market effects' translate into a positive relationship between a region's labour force and the wage rate. A number of mechanisms have been explored in the literature. One is matching; the larger the pool of workers that a firm can access the more likely it is to be able to find the exact skills that suits its needs (Helsley and Strange, 1990; Amiti and Pissarides, 2005). Another is risk sharing; if firms are subject to idiosyncratic shocks then a larger labour market exposes workers to less risk by increasing the probability of re-employment if they are made redundant (Krugman (1991b). Perhaps the most important argument is that a large labour market increases the incentives for workers to undertake training. In a small market, workers who acquire specialist skills may be 'held-up' by monopsonistic employers, in which case there is no incentive for them to invest in skills. The presence of a large number of potential employers removes this threat of opportunistic behaviour, and thereby increases the incentives for skill acquisition (Matouschek and Robert-Nicoud 2005).

5.3 Knowledge spillovers and externalities:

The third set of arguments underlying a positive *EE* relationship is something of a catch-all residual category. Knowledge spillovers are easier between proximate firms than remote ones. The mechanism may be labor mobility, face-to-face social contact between workers, or ability to observe the practices of other firms. Such effects are particularly important in innovation intensive activities, and a large literature points to the resulting spatial concentration of innovative activities (e.g. Audretsch and Feldman 2004). Location specific knowledge spillovers also arise as firms learn about the characteristics of knowledge transmission (Glaeser 1999). Much work (including Jacobs 1969) focuses on cities as centres of innovative activity. These approaches are surveyed in Duranton and Puga (2004) and their implications for urban systems analysed in Abdel-Rahman and Anas (2004).

Of course, high levels of population and employment create some negative externalities as well as positive ones, most obviously because of congestion. The magnitude and balance between these effects is thus an empirical matter. This is the issue to which we now turn.

5.4 The EE relationship: The empirical evidence:

Empirically, what do we know about the *EE* relationship? That is, what can the data tell us about the relationship between the size of a city/region and the productivity of firms and hence the level of earnings within the locality?

One approach to this problem is to estimate the production function directly with total factor productivity (the value of production for given inputs of labour and capital) modelled as a function of variables related to city/region size, for example population/employment density. The unit of observation might be the firm, the city-industry or the city. Regardless, in general terms, this involves estimating a relationship of the form:

Value of production = *f* (labour, skills, capital, density, diversity, specialisation, sector specific variables, city fixed effects, time dummies) (7)

Following this approach yields a number of findings. The net effect of density on total factor productivity is estimated to be positive (Rosenthal and Strange 2004, p. 2133). This observation confirms that cities exist because there are productivity advantages when economic agents are located together (agglomeration economies) that more than offset the congestion costs associated with higher densities. That is, cities are *not* just about shared public goods and/or rent seeking. Furthermore, cities are different sizes because the extent of agglomeration economies varies across different production activities (Rosenthal and Strange 2004, p. 2134). Some production activities (particularly in high-tech sectors) benefit from being in places where lots of similar activity is taking place (localisation economies) while others benefit from locating in a diverse environment (urbanisation economies) (Henderson 2003). The largest cities tend to be diversified, while smaller cities are more specialised. (Duranton and Puga 2000, 2001).

The major drawback of the 'production function' approach is that it is very demanding in terms of data. Obtaining measures of capital stock at the city/region level is particularly problematic. An alternative approach is to work directly with the relationship between wages and employment/population. In this case, the general form of the relationship is:

Wage = *f* (labour, skills, density, diversity, specialisation, sector specific variables, city fixed effects, time dummies) (8)

Comparing this to the specification in equation (7), the only differences are that the wage is the dependent variable and measures of capital stock are no longer included on the right-hand side. This relationship has sound theoretical foundations providing the price of capital is the same in all cities/regions.

Looking across this literature, there is strong evidence of a positive relationship between earnings and employment/population at the city/region level. Doubling population density increases wages by between 3 and 6%. For evidence relating to the US, see Ciccone and Hall (1995); for selected EU countries, see Ciccone (2002); for the UK, see Rice, Venables and Pattachini (2006); for France, see Combes, Duranton and Gobillon (2004). There is some evidence of direct interaction across neighbouring locations.[9] Ciccone identifies an additional effect of approximately 3% from the employment density of neighbouring regions. However, these effects appear to decline steeply with distance. In the case of the UK, for example, Rice et al (2006) find no evidence of effects between locations more than 80 minutes apart in terms of travel time. Decreasing travel time or distance by 10% between regions results in predicted productivity gains of 1.14% and 0.24% in the UK and France, respectively.

Analysis of relationships such as (7) and (8) requires that we control carefully for differences in the skill composition of the workforce across locations. Evidence from Rice et al (2006) suggests that one third of UK regional inequalities in earnings

[9] This direct interaction is in addition to the equilibrium interactions of our model.

can be explained by differences in skill composition. For France, Combes et al (2004) suggest a somewhat larger effect, with around 50% of regional inequality explained by differences in skills, while urbanisation and localisation economies explain about 20%. Controlling for skill is inherently problematic because of sorting effects. Formal measures of training or occupation only capture part of workers' abilities, and if there is a positive correlation between the unmeasured part of ability and workers' decisions to live in large urban areas, then econometric work will tend to overstate the productivity effects of cities.

The approaches reviewed above focus on the static aspects of agglomeration economies. There is another strand of literature on agglomeration economies concerned with the dynamics of urban growth. A simple specification to study the role of dynamic externalities can be derived by assuming local externalities affect the growth rate, rather than the level of productivity: i.e.

Growth in the value of production between period *t-s* and period *t* = *f* (density at time *t-s*, diversity at time *t-s*, specialisation at time *t-s*) (9)

This approach is often applied to long-run growth rates, that is with *s* quite large and allowing the effects to differ across sectors. Unfortunately, data on production by regions and sectors is scarce and rarely available at different points in time. For these reasons, many studies analyse the growth of employment instead of the growth in production (Glaeser, Kallal, Scheinkman, and Schleifer, 1992, Henderson, Kuncoro and Turner, 1995). More recent studies use time series econometrics to estimate full dynamic models of employment (for example: Henderson, 1997; Combes, Magnac and Robin, 2004).

In summary, the existing evidence suggests that, at least for some range of city sizes, the *EE* relationship may well be upward sloping, $\eta_w > 0$.[10] A number of econometric difficulties remain to be fully resolved (e.g. endogeneity, the need to control for compositional characteristics) but the recent literature that has begun to address these issues still points to a positive relationship.

[10] Au and Henderson (2004) provide evidence that productivity effects in China follow an inverted-U so that productivity (and hence wages) are only increasing over some range of city sizes.

6. The Cost of Living and Migration

While much of the research effort has focussed on the relationship between employment and earnings, our analytical discussion makes clear that the role of migration and the relationship between the cost-of-living and population size are also important in shaping economic linkages across space. In this section we briefly review the evidence relating to these although, relative to work on the *EE* curve, the literature tends to be more eclectic in terms of methodology and applications.

6.1 The cost-of-living and the HH relationship

The cost-of-living depends on product prices, the price of land and housing, and additional costs such as commuting. The original new economic geography models (Krugman 1991a and following) ignored housing, rents, and intra-regional transport, so the costs of living are determined only by the price of goods. This tends to generate a negative *HH* relationship – regions with a large population have lower living costs – the opposite of the relationship illustrated in the preceding figures. There are three related arguments for this. The first is that a large region tends to have lower transport costs because a smaller proportion of goods are imported. The second is that there is a wider range of products on offer (think of non-tradable goods such as restaurants) and, given that people like variety, these have the effect of bringing down the cost of producing utility (and hence the true cost of living index). The third is that competition between firms is more intense in a dense area of activity, and this brings down prices for consumers.

While these product market factors are important, empirically they often seem to be outweighed by considerations relating to land and housing, aspects that were crucial in earlier work on urban and regional economics.[11] A positive *HH* relationship could be based on simply postulating an upwards-sloping supply of land for housing, so increasing population raises its price. An alternative, on which much of the urban economics literature is based, supposes that the price of each marginal unit of land in the area is constant. However, commuting costs within each city create a premium on city centre land. Thus, in the simplest model, all employment takes place in the 'central

[11] The first attempt to combine Krugman's model with increasing land prices was Helpman (1998).

business district' (CBD). Urban land rents adjust so that, in equilibrium, all individuals are indifferent about where in the city they are housed and, since commuting costs increase with distance from the CBD, land rents must decline with distance. What does this imply for the relationship between city size and cost of living, that is the *HH* schedule? In the simplest case (a linear city, equal size residential lots and commuting costs proportional to distance) the cost of urban living (rent plus commuting cost) increases with the square of population. More generally, the cost is increasing and convex.

The standard empirical model of the housing market consists of a demand equation that determines house prices in the short run as a function of the housing stock, population, income per household, interest rates etc; a supply equation that determines the supply of new housing; and an adjustment equation determining how the stock of houses adjusts over time as new houses are completed. Within this framework, the long run impact of population change on house prices depends on the price elasticities of housing demand and supply[12]. A recent review of evidence for the UK suggests that the price elasticity of demand for housing is of the order of -0.5. If the supply of housing is price inelastic then the elasticity of house prices with respect to the number of households is estimated to be between 1.7 and 2.5 (Meen and Andrews, 1998; Meen 1998, 2002). The larger is the supply elasticity the less responsive are house prices to population changes in the long run (i.e. the steeper is the *HH* curve). For example, increasing the supply elasticity to 1 reduces the elasticity of house prices with respect to population changes to less than 1. However, given that new housing construction is small in relation to the existing housing stock, prices may be very slow in achieving their long run equilibrium values. Evidence from model simulation suggests that with a supply elasticity of around 1, house prices would remain significantly above their long run equilibrium value some 40 years after an initial increase in population (Meen 1998).

Recent estimates of the price elasticity of housing supply vary widely across countries. For the UK, Barker (2004) reports a range of estimates for the price elasticity of supply of new housing from about 0.3 to 1.0; with evidence of substantial variation in supply elasticities across regions (see e.g. Meen, 2003). Swank, Kakes and Tiemen (2002) provide international comparisons of the price elasticity across countries,

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[12] The elasticity of house prices with respect to population is given by $\varepsilon_L^d / (\varepsilon^d + \varepsilon^s)$ where ε_L^d is the elasticity of demand for housing with respect to population ; ε^d is the price elasticity of demand for housing and ε^s is the price elasticity of supply of housing.

ranging from lows of 0.3 for the Netherlands and 0.5 for the UK to highs of 1.4 for the US and 2.1 for Germany. The constraints imposed by the planning system play a major role in determining supply elasticity. For example, the land use planning system in the UK determines the supply of land essentially independently of price. Since a central intention of this planning is to contain urban development by fixing growth boundaries, the supply of space is quasi-fixed. While it is possible to increase the density of occupation of a given land area, evidence suggest that this margin of flexibility has not been sufficient to prevent the elasticity of supply of housing from decreasing in the UK between the early and later parts of the 20th century as supply constraints became binding (Malpezzi and Maclennan, 2001). A number of studies of US regional housing markets have identified low supply elasticity of housing due to land use regulations as an important factor behind rapid housing price increases in a number of US metropolitan areas, most notably San Francisco, Boston and Los Angeles (Evenson 2002; Glaeser and Gyourko, 2003; Glaeser et al., 2005). Gyourko, Mayer and Sinai (2006) point to the role of heterogeneity in locations' endowment of amenities, and the likelihood that supply elasticities might be particularly low in desirable locations. Combining this with widening income inequality creates rising house prices in desirable 'superstar cities'.

Finally, what of the possibility that house prices and hence the cost of living are directly affected by changes in wages? The consensus in the empirical literature is that the long run effect on house prices of changes in income per household (i.e. wages) is at least as great as that of changes in the number of households (Cheshire, 2008). However, as demonstrated in section 4, with a positive earnings-employment relationship, productivity shocks give rise to positively correlated movements in house prices and wages, even in the absence of any direct causal link between wages and house prices. The possible joint endogeneity of house prices and household incomes and the implications for estimation of the income elasticity of demand for housing are issues which have been hitherto ignored in the literature. This discussion serves to highlight, once again, that the reduced form relationships are helpful for simplifying and thinking through the nature of linkages but that further progress will require a more detailed understanding of the exact nature of the underlying processes.

6.2 Migration and the MM relationship:

The third key ingredient is migration. How mobile is labour between regions? Answering this question is difficult, because it is quite possible that very high degrees of potential mobility coexist with low levels of actual mobility, precisely because housing prices are acting to equilibrate flows, making households indifferent between living in different regions.

On average 1 percent of households migrate between major regions of the UK each year, with net outflows from London, the North East, North West and West Midlands and net inflows into the South East, South West and East (HM Treasury, 2001). The empirical evidence for the UK supports the contention that workers' location decisions are responsive to real wage differentials. A body of econometric evidence has built up showing that high relative wages significantly increase net migration into a region; while high relative house prices discourage it (see Murphy, Muellbauer and Cameron, 2006 for a recent review). There is also evidence of a migration/commuting trade-off (Jackman and Savouri, 1996; Murphy, Muellbauer and Cameron, 2006). For contiguous regions, where relatively cheap commuting offers an alternative to migration, the location decision tends to be more strongly influenced by housing price differentials and less influenced by wage differentials. The reverse is true if commuting is more costly. This suggests that migration in the UK will move the economy along the *ZZ* curve towards the intersection with the *MM* curve where real incomes are equalised.

One important caveat is in order, however. Within these average mobility rates there are large variations in migration rates between skill groups, with higher skill groups displaying a significantly higher propensity to move between locations than their lower skilled counterparts. A number of possible explanations have been offered for this difference. The costs of migration – particularly those associated with transacting in the housing market – may be relatively greater for lower skilled workers (Hughes and McCormick, 2000). Lower skilled workers tend to rely more heavily on local informal networks of contacts for information regarding the job market. Finally, the external benefits associated with 'thick' labour markets produce clustering of high skilled jobs. Workers sort by location whereby more able workers group themselves in particular locations where there are the largest economic returns. In particular contexts, we may want to allow for these differences in mobility across different groups.

7. Conclusion

We have argued that three relationships play a key role in determining the economic linkages between cities. The first concerns the link between local employment and earnings, the second the link between local employment and the cost of living and the third the migration response to differences in real wages between locations. These three relationships provide the basis for a graphical framework for analysing the linkages between cities or regions. We can use this framework to draw out the implications of existing theoretical models for the linkages between cities, even though these models are not explicitly concerned with these linkages. Perhaps more importantly for policy purposes, this graphical framework allows us to consider when gains in one city or region spillover positively or negatively to other areas.

The simple reduced framework set out in sections 3 and 4 highlights the importance of the nature of the relationship between employment and earnings in determining the direction of spillovers between cities/regions. If the relationship between employment and earnings is negative then complementarity applies, so that a positive productivity shock to one city creates positive spillovers to other cities as economic adjustment dampens and disperses the impact of shocks. This outcome is reversed if the relationship between employment and earnings is positive as a result of agglomeration economies. In these circumstances cities or regions are in a competitive relationship, and the process of adjustment to shocks tends to amplify the gains to one area. If workers are perfectly mobile then migration flows ensure that these changes in nominal earnings do not translate in to differences in real earnings (because the cost of living adjusts to offset the earnings differential). If this is not the case, workers in the other cities can see their real standard of living decline.

Given their importance, what do we know about the nature of the three key relationships? A priori theoretical reasoning cannot help us choose between the different possibilities and we must turn instead to empirical evidence. We have reviewed the evidence on all three relationships, with a particular focus on the earnings-employment relationship where, arguably, our degree of ignorance is greatest. We reach a number of conclusions. First, the nature of the housing market in the UK means that the cost of living increases quickly with rising employment. Second, and consistent with our theoretical framework, this cost of living effect implies small migration flows between cities, even though the evidence suggests that many workers are quite mobile and respond to real earnings differentials between places. It is likely that commuting between cities partially substitutes for the migration that would occur in the absence of cost of living effects. Third, the employment-earnings relationship may be upward sloping meaning that population movement between cities or regions tends to widen earnings differentials, but this relationship is almost certainly sector and location specific.

This specificity makes it hard to reach general conclusions about magnitudes, but does allow us to generalise about the direction of changes induced by the linkages between regions. Specifically, the positive employment relationship, at least for some ranges of city sizes points to the possibility of a competing relationship. As we have seen this means that an initial positive shock to, say, productivity in one region will get magnified as workers migrate from other regions. However, as workers migrate, living costs are driven up. Where housing supply is relatively inelastic, as in the UK, this may be associated with quite small movements in population.

Do the resulting changes in employment, wages and cost of living matter? For mobile workers the answer is almost certainly not. Willingness to migrate ensures that real wages are equalized independent of location choice. In the UK, at least, this suggests that differences in wages and house prices are not a major issue for young skilled graduates who are highly mobile. For lower skilled workers and others who, for one reason or another, may be immobile, the effects are more complicated. In the expanding region, these workers may see increases in housing costs outweighing increases in wages if agglomeration effects are larger for skilled than unskilled (which empirically may be the case). In contracting regions the opposite effect may occur. These differences will also play out differently depending on the pattern of home ownership with implications for wealth as house prices change. Clearly, further work is needed, but our theoretical model, coupled with the limited empirical evidence at least provides a starting point for thinking about these issues.

Theoretical appendix

There are K regions, and region i has labour forces L_i, with $\Sigma_i L_i = L$. Wage rates are w_i costs of living are h_i. We give general forms of relationships and their iso-elastic version. The earnings employment relationship is

$$w_i = a_i w(L_i) = a_i L_i^{\eta_w}, \tag{A1}$$

where η_w is the elasticity of earnings with respect to employment. The cost of living relationship is

$$h_i = h(L_i / b_i) = (L_i / b_i)^{\eta_h}, \tag{A2}$$

We think of this principally in terms of the housing market, so an increase in b_i represents an increase in region i housing stock. η_h is the elasticity of the cost of living with respect to population. The migration relationship is

$$c_i w_i / h_i = c_j w_j / h_j, \quad \text{for all pairs of locations, } i, j, \tag{A3}$$

where c_i is a shift parameter, and an increase would represent an increase in the amenity value of living in region i.

Using A1 and A2 in A3 equilibrium values of L_i satisfy,

$$\frac{c_i a_i w(L_i)}{h(L_i / b_i)} = \frac{c_j a_j w(L_j)}{h(L_j / b_j)}, \quad \text{for all pairs of locations, } i, j,$$

In iso-elastic form: $$c_i a_i b_i^{\eta_h} L_i^{\eta_w - \eta_h} = c_j a_j b_j^{\eta_h} L_j^{\eta_w - \eta_h} \tag{A4}$$

Now consider a shock to a particular region, S. Changes in employment across all regions sum to zero, and if we assume that all regions are initially symmetric this condition implies

$$\hat{L}_S + (K-1)\hat{L}_j = 0 \tag{A5}$$

where $\hat{L}_j$ is the change in each region other than S. Now differentiating A4 and using A5 we derive, in the neighbourhood of the symmetric equilibrium:

$$\hat{L}_S = \left(\frac{K-1}{K}\right)\left(\frac{\hat{c}_S + \hat{a}_S + \hat{b}_S \eta_h}{\eta_h - \eta_w}\right), \qquad \hat{L}_j = \left(\frac{-1}{K}\right)\left(\frac{\hat{c}_S + \hat{a}_S + \hat{b}_S \eta_h}{\eta_h - \eta_w}\right) \tag{A6}$$

The effects of various shocks on other endogenous variables are given by using A6 in A1 – A3:

Productivity shock in S: $\hat{a}_S$

$$\hat{w}_S = \hat{a}_S + \left(\frac{K-1}{K}\right)\left(\frac{\hat{a}_S \eta_w}{\eta_h - \eta_w}\right), \qquad \hat{w}_j = \left(\frac{-1}{K}\right)\left(\frac{\hat{a}_S \eta_w}{\eta_h - \eta_w}\right)$$

$$\hat{h}_S = \left(\frac{K-1}{K}\right)\left(\frac{\hat{a}_S \eta_h}{\eta_h - \eta_w}\right), \qquad \hat{h}_j = \left(\frac{-1}{K}\right)\left(\frac{\hat{a}_S \eta_h}{\eta_h - \eta_w}\right)$$

Real income: $\hat{w}_S - \hat{h}_S = \hat{a}_S / K = \hat{w}_j - \hat{h}_j$

Increase house supply in S: $\hat{b}_S$

$$\hat{w}_S = \left(\frac{K-1}{K}\right)\left(\frac{\hat{b}_S \eta_h \eta_w}{\eta_h - \eta_w}\right), \qquad \hat{w}_j = \left(\frac{-1}{K}\right)\left(\frac{\hat{b}_S \eta_h \eta_w}{\eta_h - \eta_w}\right),$$

$$\hat{h}_S = \frac{-\eta_h \hat{b}_S}{(\eta_h - \eta_w)}\left(\frac{\eta_h - K\eta_w}{K}\right), \qquad \hat{h}_j = \frac{-\eta_h \hat{b}_S}{\eta_h - \eta_w}\left(\frac{\eta_h}{K}\right)$$

Real income: $\hat{w}_S - \hat{h}_S = \eta_h \hat{b}_S / K = \hat{w}_j - \hat{h}_j$

Increase amenity in S: $\hat{c}_S$

$$\hat{w}_S = \left(\frac{K-1}{K}\right)\left(\frac{\hat{c}_S \eta_w}{\eta_h - \eta_w}\right), \qquad \hat{w}_j = \left(\frac{-1}{K}\right)\left(\frac{\hat{c}_S \eta_w}{\eta_h - \eta_w}\right)$$

$$\hat{h}_S = \left(\frac{K-1}{K}\right)\left(\frac{\hat{c}_S \eta_h}{\eta_h - \eta_w}\right), \qquad \hat{h}_j = \left(\frac{-1}{K}\right)\left(\frac{\hat{c}_S \eta_h}{\eta_h - \eta_w}\right)$$

Real income: $\hat{w}_S - \hat{h}_S + \hat{c}_S = \hat{c}_S / K = \hat{w}_j - \hat{h}_j$

Cost of living depends on employment and wages: In the text we have modelled the cost of living as a function of employment. Some of this dependence may come via wages, so the *h*(.) function can be decomposed (using the iso-elastic case) to $h_i = (L_i / b_i)^{\varepsilon_h} \left(a_i L_i^{\eta_w}\right)^{\varepsilon_w}$, where ε_w is the elasticity of the cost of living with respect to the wage rate, ε_h is the elasticity of the cost of living with respect to labour force (conditional on the wage rate), and η_h is the sum of the partial effects, $\eta_h = \varepsilon_h + \eta_w \varepsilon_w$. This generalization makes no difference to the figures or qualitative analysis, but comparative statics are now based on equation A4' with A5, where

$$c_i a_i^{1-\varepsilon_w} b_i^{\varepsilon_h} L_i^{\eta_w(1-\varepsilon_w)-\varepsilon_h} = c_j a_j^{1-\varepsilon_w} b_j^{\varepsilon_h} L_j^{\eta_w(1-\varepsilon_w)-\varepsilon_h}. \qquad \text{(A4')}$$

The implications of a direct effect of wages on the cost of living, $\varepsilon_w > 0$, for the analysis of sections 3 and 4 depends on the sign of η_w. If $\eta_w < 0$ (the complementary case) then $\varepsilon_w > 0$ makes η_h smaller and the *HH* curve steeper. As noted in (4.1) a steeper *HH* curve means that a shock leads to relatively large movements in population and small changes in costs of living and wages. With $\eta_w > 0$ (the competing case), $\varepsilon_w > 0$ increases the sum of the partial effects, η_h. The *HH* curve depicted in figures 1-6 tends to be flatter, and small movements in population lead to relatively large changes in living costs of living. Notice that the dividing line between competing and complementary cases, that is where $\eta_w = 0$, is unaffected by allowing an effect through $\varepsilon_w > 0$. However, the value of ε_w does matter for the emergence of a divergent regime. Divergence requires that $\eta_w > \eta_h = \varepsilon_h + \eta_w \varepsilon_w$, which is not possible if $\varepsilon_w \geq 1$.

References

Abdel-Rahman, H.M. and A. Anas (2004), "Theories of systems of cities" in J.F. Thisse and J. V. Henderson (eds) *Handbook of Urban and Regional Economics* (vol 4), North Holland.

Amiti, M. and C. Pissarides (2005) 'Trade and Industrial Location with Heterogeneous Labour", *Journal of International Economics,* 67(2), 392-412.

Au, C. and V. Henderson (2004) "How migration restrictions limit agglomeration and productivity in China", Processed Brown University.

Audretsch, D. and M. Feldman (2004) 'The geography of innovation' in in J.F. Thisse and J. V. Henderson (eds) *Handbook of Urban and Regional Economics* (vol 4), North Holland.

Barker, K. (2004) Review of Housing Supply: Securing our Future Housing Needs: Interim Report – Analysis, London: HMSO.

Cheshire, P. (2008), 'Reflections on the nature and policy implications of planning restrictions and housing supply'. Forthcoming *Oxford Review of Economic Policy*.

Ciccone, A. (2002), "Agglomeration Effects in Europe", *European Economic Review*, 46, 213-227.

Ciccone, A., and R. Hall (1996), "Productivity, and the Density of Economic Activity", *American Economic Review*, 86, 54-70.

Cingano, F. and F. Schivardi (2004), "Identifying the Sources of Local Productivity Growth", *Journal of the European Economic Association* 2(4): 720-742.

Combes, P.-P., G. Duranton, and L. Gobillon (2003), "Wage Differentials across French Local Labour Markets: Endowments, Skills, and Interactions", processed LSE, London.

Combes, P.-P., and M. Lafourcade (2003), "Core-Periphery Patterns of Generalized Transport Costs: France, 1978-1998", CEPR discussion paper, no. 3958.

Combes, P.-P., T. Magnac, and J.-M. Robin (2004), "The Dynamics of Local Employment in France", *Journal Urban Economics* 56 pp. 217-243

Combes, P.P and H. G. Overman (2004) 'The Spatial Distribution of Economic Activities in the European Union'. J.F. Thisse and Vernon Henderson (eds) *Handbook of Urban and Regional Economics* (vol 4), North Holland.

Dixit, A. and J. Stiglitz (1977), "Monopolistic Competition, and Optimum Product Diversity", *American Economic Review,* 67, 297-308.

Duranton, G. and D. Puga (2000), "Diversity and specialisation in cities: Why, where and when does it matter?" *Urban Studies* 37(3):533–555.

Duranton, G. and D. Puga (2001), "Nursery cities: Urban diversity, process innovation, and the life cycle of products" *American Economic Review* 91(5):1454–1477.

Duranton, G. and D. Puga (2004), "Micro-foundations of urban agglomeration economies" in J.F. Thisse and J. V. Henderson (eds) *Handbook of Urban and Regional Economics* (vol 4), North Holland.

Duranton, G. and D. Puga (2005), "From sectoral to functional urban specialization", *Journal of Urban Economics,* 57(2): 343-370.

Evenson, B. L., (2002) "Understanding House Price Volatility: Measuring and Explaining the Supply Side of Metropolitan Area Housing Markets", Illinois State University Department of Economics Working Paper. http://ssrn.com/abstract=352400

Fujita, M., P. R. Krugman, and A. J. Venables (1999), *The Spatial Economy: Cities, Regions and International Trade*, MIT press.

Glaeser, E. (1999), "Learning in cities", *Journal of Urban Economics*, 46(2):254-277.

Glaeser, E. and J. Gyourko (2003), "The impact of building restrictions on housing affordability", *Federal Reserve Bank of New York Economic Policy Review, June.*

Glaeser, E., J. Gyourko and R. Saks (2005), "Why have housing prices gone up?", *Harvard Institute of Economic Research Discussion Paper,* No.2061.

Glaeser, E., H. Kallal, J. Sheinkman, and A. Schleifer (1992), "Growth in Cities", *Journal of Political Economy*, 100, 1126-1152.

Gyourko, J., C. Mayer, and T. Sinai. (2006) "Superstar cities" NBER Working Paper No. 12355

Hanson, G. (2005), "Market Potential, Increasing Returns, and Geographic Concentration", *Journal of International Economics*, forthcoming.

Helpman, E. (1998), "The size of regions". In D. Pines, E. Sadka, and I. Zilcha (eds.) *Topics in Public Economics. Theoretical and Applied Analysis.* New York, NY: Cambridge University Press, 33–54.

Helsley, R. W. and W. C. Strange. (1990). "Matching and agglomeration economies in a system of cities". *Regional Science and Urban Economics* 20(2):189–212.

Henderson, J. V. (1974), “The sizes and types of cities” *American Economic Review* 64(4):640–656.

Henderson, J. V. (1988), *Urban development: Theory, fact, and illusion*, Oxford University Press, New-York.

Henderson, J. (1997) “Externalities and industrial development”, *Journal of Urban Economics*, 42, 449-470.

Henderson, J.V. (2003), “Marshall’s Scale Economies,” *Journal of Urban Economics* 53: 1-28.

Henderson, V., A. Kuncoro, and M. Turner (1995), “Industrial Development in Cities”, *Journal of Political Economy*, 103, 1067-1152.

Hughes, G.A. and B. McCormick, (2000) *Housing Policy and Labour Market Performance,* Report to the Office of the Deputy Prime Minister.

Hughes, G.A. and B. McCormick, (1987), “Housing markets, unemployment and labour market flexibility in the UK”, *European Economic Review* 31, 615-645.

Irmen A. and J.-F. Thisse, (1998) “Competition in multi-characteristics spaces: Hotelling was almost right”, *Journal of Economic Theory* 78, 76-102.

Jackman, R and Savouri, S. (1996), “Regional Migration and the Hiring Function: An Examination of Distance and Contiguity Effects in Great Britain, 1975-92”, in Schettkat, R. (ed), *The Flow Analysis of Labour Markets.* Routledge, London.

Jacobs, J. (1969), *The Economy of Cities.* Random House: New York.

Jaffe, A. B. M. Trajtenberg, and R. Henderson (1993), “Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations,” *Quarterly Journal of Economics* 108: 577-598.

Krugman, P. R. (1991a), “Increasing returns and economic geography”. *Journal of Political Economy* 99(3):484–499.

Krugman, Paul R. (1991b). *Geography and Trade*. MIT Press, Cambridge, MA.

Krugman, P. R. and A. J. Venables (1995), “Globalization and the inequality of nations”. *Quarterly Journal of Economics* 110(4):857–880.

Leamer, E. E. & J. Levinsohn, (1995) "International trade theory: The evidence," in G. Grossman and K. Rogoff (eds), *Handbook of International Economics*, North Holland, Amsterdam

Malpezzi, S. and D. Maclennan (2001) ‘The Long-Run Price Elasticity of New residential Construction in the United States and the United Kingdom’, *Journal of Housing Economics*, 10, 278-306.

Marshall, Alfred (1890) *Principles of Economics,* London: Macmillan and Co., Ltd.

Matouschek, N. and F. Robert-Nicoud, (2005), "The Role of Human Capital Investments in the Location Decision of Firms", *Regional Science and Urban Economics* 35(5), pp. 570-83.

McCormick, B. (1997) "Regional unemployment and labour mobility", *European Economic Review*, 41,581-589.

Meen, G. and M. Andrew (1998), "*Modelling Regional House Prices: A Review of the Literature"* Report to the Department of the Environment, Transport and the Regions.

Meen, G. (2003), *Regional Housing Supply Elasticities in England,* Centre for Spatial and Real Estate Economics, University of Reading Business School (October).

Mion, G. (2004), "Spatial Externalities and Empirical Analysis: The Case of Italy", *Journal of Urban Economics* 56(1), 97-118.

Murphy, A., J. Muellbauer and G. Cameron (2006), "Housing market dynamics and regional migration in Britain". Mimeo, University of Oxford, August 2006.

Rice, P. and A.J. Venables (2003), "Equilibrium Regional Disparities: Theory and British Evidence", *Regional Studies,* 37 (6/7), 675-687

Rice, P., A.J. Venables and E. Patacchini (2006), "Spatial Determinants of Productivity: Analysis of the Regions of Great Britain", *Regional Science and Urban Economics,* 36 , 727-752.

Rosenthal, S. S., and W. C. Strange (2004) "Evidence on the Nature and Sources of Agglomeration Economies" in J.F. Thisse and J.V. Henderson, *Handbook of Regional and Urban Economics,* Vol. 4, North-Holland.

Sheppard, S. (1999) 'Hedonic Analysis of Housing Markets', in *Handbook of Regional and Urban Economics* Vol. III: *Applied Urban Economics*, edited by P.C.Cheshire and E.S. Mills, Elsevier Science Publishers.

Swank, J., Kakes, J. and Tieman, A. (2002) 'The Housing Ladder, Taxation, and Borrowing Constraints:' DNB Staff Reports No. 9.

Tabuchi, T. and J.-F. Thisse. (2002), "Taste heterogeneity, labour mobility and economic geography". *Journal of Development Economics* 69(1):155–177.

Venables, A.J. (1996) 'Equilibrium Locations of Vertically Linked Industries', *International Economic Review*, 37, 341-59.

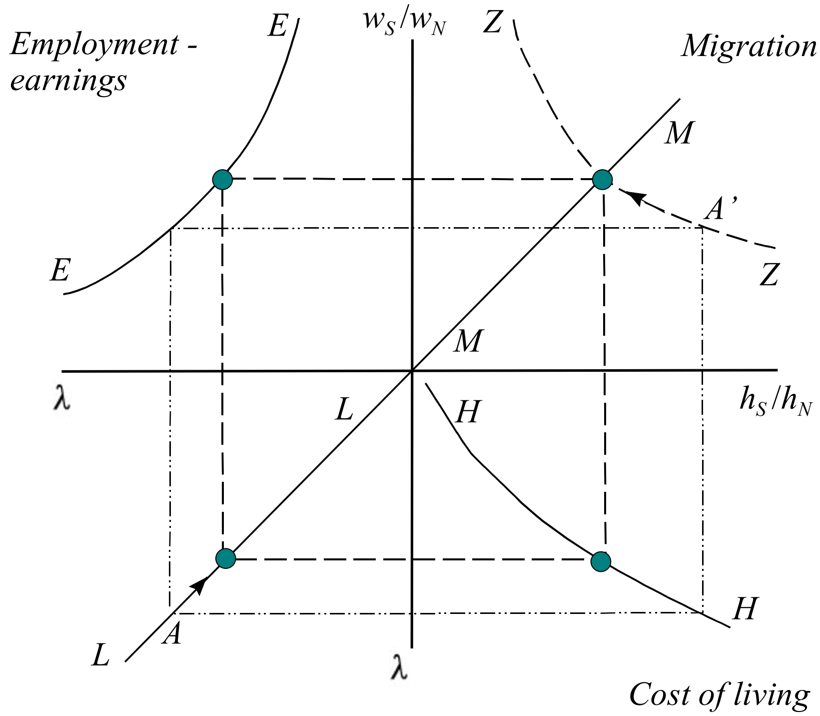

Figure 1: Complementary regions; $\eta_h > 0,\ \eta_w < 0$

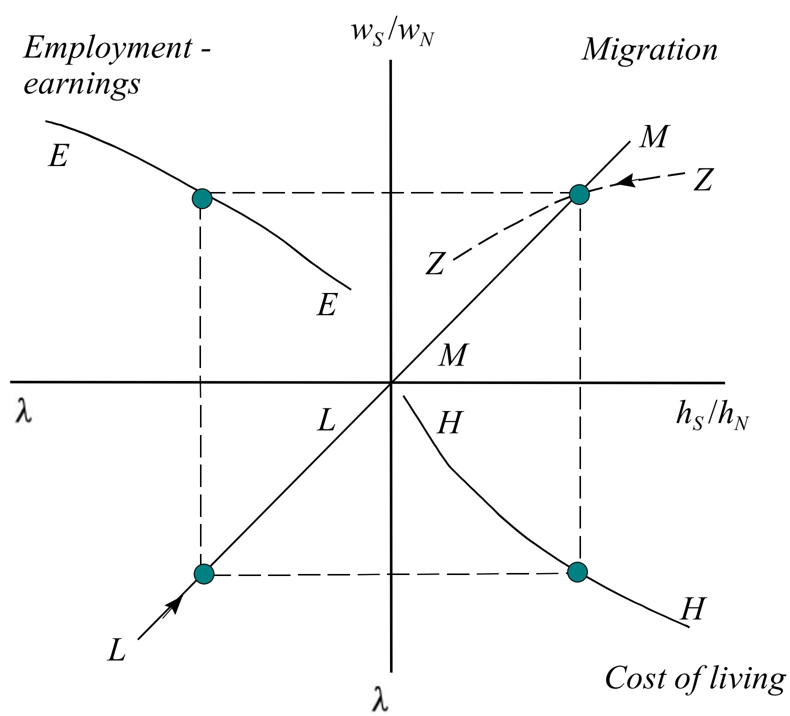

Figure 2: Competing regions; $\eta_h > \eta_w > 0$

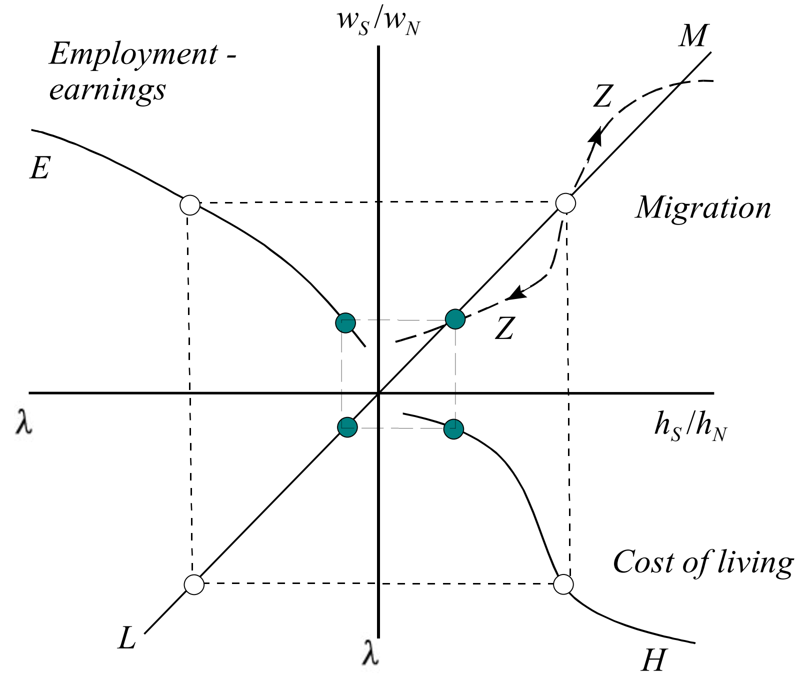

Figure 3: Divergent regions; $\eta_w > \eta_h > 0$

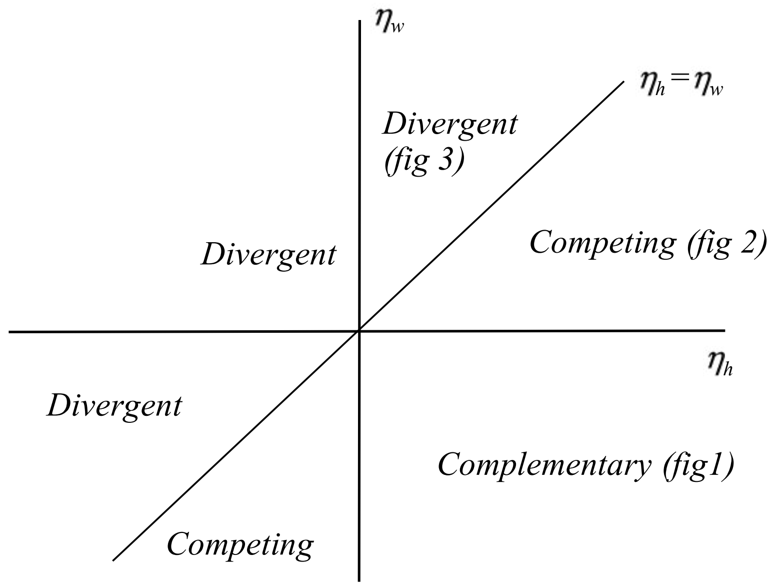

Figure 4: Regime summary

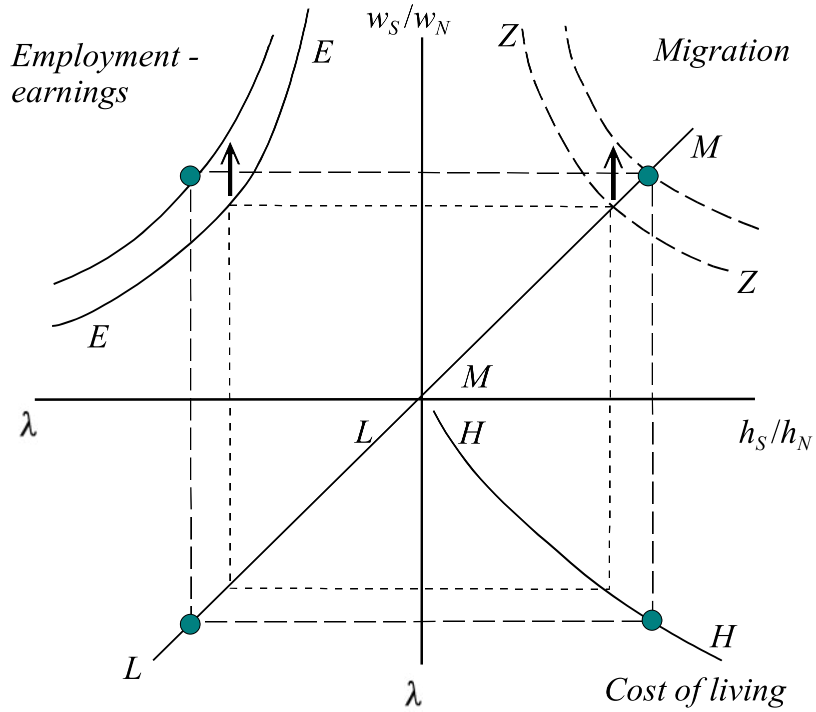

Figure 5a: Productivity improvement in S: complementary regions

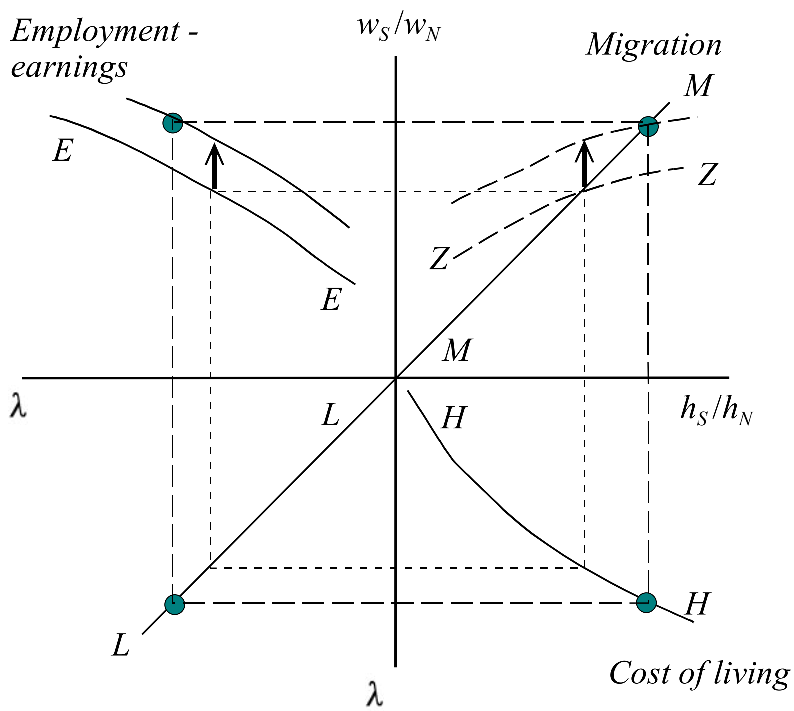

Figure 5b: Productivity improvement in S: competing regions

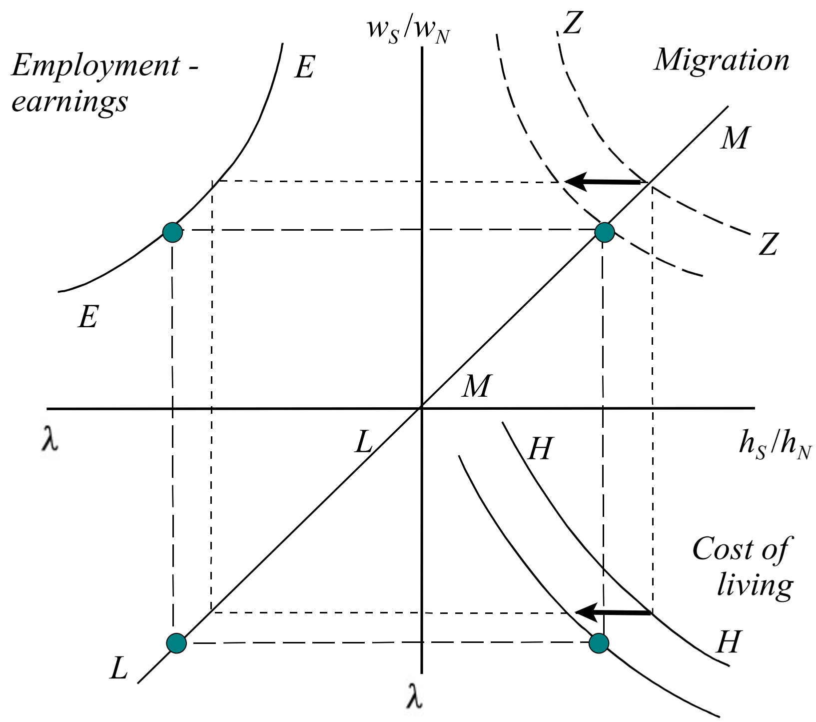

Figure 6a: Increased land in S: complementary regions

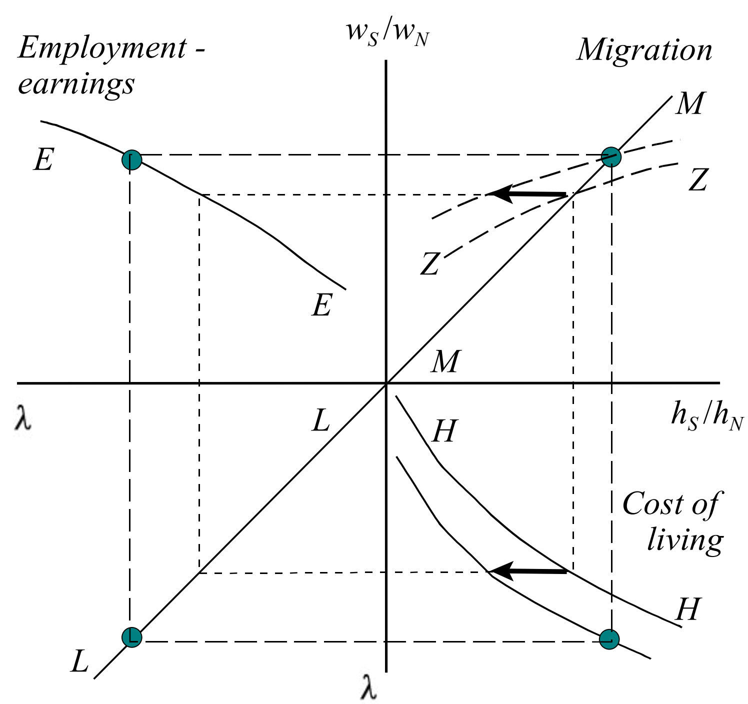

Figure 6b: Increased land in S: competing regions