The mix of public inputs under tax competition

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Abstract

This paper analyzes how the use of taxation on mobile factors affects the mix of “factor-specific public inputs” set by regional governments (e.g., manpower training for labor; infrastructure for capital). These inputs are defined such that they have impacts analogous to the Harrod-neutral and Solow-neutral technical change. Regional governments provide two types of public input that, respectively, complement immobile and mobile factors. It is shown that the nature of the expenditure mix depends on the elasticity of substitution between these factors. Too much tax revenue is spent on public input complementing mobile (immobile) factors if the elasticity is more than (less than) one, but the mix of these inputs is efficient if the elasticity is equal to one.

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1. Introduction

Expenditures on productive activities, including human capital formation, research and development, land development and infrastructure account for a non-trivial share of public budgets. These public services can be regarded as independent variables of production functions; that is, public inputs. Although there are disputes in the context of empirical analyses over the impacts of these expenditures on productivity and economic growth, it is
well known that regional governments often use them as policy instruments for regional development and attracting business investment. ¹

The influence of competition for mobile factors on policy-making is the main subject of the tax competition literature. Several papers have considered public input provision in this context. Examples include Zodrow and Mieszkowski [14], Keen and Marchand [7], Matsumoto [9–11]. By using models with a single public input, these papers study overall public expenditure on productive activities. Given that regional governments provide public inputs of many kinds, however, it is interesting to investigate how tax revenues are allocated among them. This paper demonstrates that the use of taxation on mobile factors distorts the mix of public inputs. ² A complex conceptual problem associated with this study will be how those inputs are classified. (The same problem applies to models with public goods benefiting residents). One possible approach, which is this paper’s concern, is to analyze the mix of “factor-specific” public inputs that complement particular private factors of production (e.g., manpower training for labor, infrastructure for capital). In this paper, these inputs are defined such that they have impacts analogous to the Harrod-neutral and Solow-neutral technical change. One might consider that when studying expenditures or programs aimed at particular industries or firms, the mix of “industry-specific” or “sector-specific” public inputs is an important policy issue. However, even in this case, in the decision-making process of detailed policy packages, government officials will have to decide what factor should be supported in those favored industries or firms. On the other hand, when analyzing policy-making on general expenditures on education and infrastructure, the concept of factor-specific public inputs is helpful in examining the composition of public budgets.

This paper uses a simple tax competition model with a single (aggregated) production sector. Regional governments provide two public inputs that, respectively, complement immobile and mobile factors. This simple model yields a very clear insight into the nature of the expenditure mix. The mix inefficiency caused by tax competition depends on the magnitude of substitutability between immobile and mobile factors. The parameters representing the impacts of public input provision on production technology do not play any important role in this paper’s qualitative analysis of which type of public input is relatively underprovided or overprovided. If the elasticity of substitution is more than (less than) one, too much tax revenue is spent on public input complementing mobile (immobile) factors. If the elasticity is equal to one, the mix of public inputs is efficient relative to equilibrium tax revenues. Still, the overall level of public expenditure is inefficiently low under tax competition.

¹ See, for example, Bartik [1,2] and Fisher [3, Chapter 22]. Gramlich [4] and Haughwout [5] include reviews of empirical studies of infrastructure policies.

² Although Keen and Marchand [7] and Matsumoto [11] investigate the composition of public expenditure under tax competition, their analyses are limited to the mix of public goods and public inputs. Konrad [8] considers the composition of education and infrastructure expenditures, which are factors of production. He focuses on the impact of labor mobility on the expenditure policy made by non-altruistic old generation. His specification of regional production functions differs from that in this paper.
2. The model

Consider a small open region where a numeraire output is produced by competitive firms using an immobile factor \((l)\) and a mobile factor \((k)\). The supply of \(l\) is exogenous while \(k\) changes due to mobility. Private firms benefit from public inputs \((G^l\) and \(G^k\)) provided by the regional government. The production function is

\[
F(\alpha(G^l)l, \beta(G^k)k).
\]

\((1)\)

\(F\) exhibits constant returns to scale with decreasing marginal products. It is assumed that \(\alpha'(G^l)\) and \(\beta'(G^k)\) are both positive. \(G^l\) and \(G^k\) represent the factor-specific public inputs complementing the immobile factor and mobile factor, respectively. The functional form in \((1)\) follows the Hillman–McMillan definition of the production function in the presence of public inputs.\(^3\) From \((1)\), the public inputs have impacts analogous to the Harrod-neutral and Solow-neutral technical change. There are two relationships between the public inputs and private factors; that is, an increase in \(G^i\) \((i = l \text{ or } k)\) affects the productivity of factor \(i\) and at the same time the effective supply of that factor, \(\alpha(G^l)l\) or \(\beta(G^k)k\), is increased. For example, manpower training will increase the effective supply of labor at a given amount of that factor. Public expenditures on infrastructure increase the regional capital stock. In the present model, the policy-induced change in the effective supply of one factor generates an indirect benefit to the other by raising its marginal product, because \(l\) and \(k\) must be complements in production under constant returns to scale technology with decreasing marginal products.

The remaining part of the model is similar to that in Matsumoto [9], which is based on the familiar Zodrow–Mieszkowski model. Public expenditure is financed by a source-based tax on \(k\). Then, profit-maximization made by competitive firms implies that

\[
\beta(G^k)F_K(\alpha(G^l)l, \beta(G^k)k) = r + t,
\]

\((2)\)

where a subscript attached to the production function represents a partial derivative \(F_K = \partial F/\partial(\beta(G^k)k)\), \(t\) is the tax rate and \(r\) is the net return on the mobile factor. The small open region takes the net return as given. Assuming that one unit of the output can be transformed into one unit of \(G^l\) or \(G^k\), the public budget constraint is

\[
tk = G^k + G^l.
\]

\((3)\)

Residents own the immobile factor in the region and a fixed amount of the mobile factor \((\bar{k})\). They spend the income obtained from these sources on consumption of the output. The regional government sets public policies to maximize residents’ consumption:

\[
\text{Max}_{G^l, G^k, t}, F(\alpha(G^l)l, \beta(G^k)k) - (r + t)k + r\bar{k}, \quad \text{s.t.} \quad (2) \text{ and } (3).
\]

\((4)\)

Because the regional government takes \(r\) as given, it effectively maximizes the returns accrued from the immobile factor.

\(^3\) See Hillman [6] and McMillan [12]. Strictly speaking, they do not examine factor-specific public inputs. (In terms of this paper’s notation, the production function in Hillman and McMillan is given by \(F(\alpha(G)l, \beta(G)k)\), where \(G\) is a public input.) This paper applies their basic definition of the production function to a study of the provision of factor-specific public inputs.
3. Public expenditure

This section derives and examines the equilibrium conditions for public expenditure. Solving (2) to obtain the demand function for \( k, k = k(t, G^k, G^l) \), and substituting it into (4), the Lagrangean of the government’s optimization problem is

\[
F(\alpha(G^l)t, \beta(G^k)t, k(t, G^l, G^k)) - (r + t)k(t, G^l, G^k) \\
+ \lambda(k(t, G^l, G^k) - G^k - G^l),
\]

(5)

where \( \lambda \) is the Lagrange multiplier. The exogenous income, \( r\bar{k} \), is omitted. Using (2), the first-order conditions for \( t, G^k \) and \( G^l \), are respectively:

\[
-k + \lambda \left( k + \frac{\partial k}{\partial t} \right) = 0,
\]

(6)

\[
\beta' F_K k + \lambda \left( \frac{\partial k}{\partial G^k} - 1 \right) = 0,
\]

(7)

\[
\alpha' F_L l + \lambda \left( \frac{\partial k}{\partial G^l} - 1 \right) = 0,
\]

(8)

where \( F_L = \frac{\partial F}{\partial (\alpha(G^l)l)} \). These conditions include policy-induced changes in the mobile tax base. Differentiating (2) gives

\[
\frac{\partial k}{\partial t} = \frac{1}{\beta^2 F_{KK}} - \frac{F_k}{\alpha \beta' F_L F_K l} \sigma,
\]

(9)

\[
\frac{\partial k}{\partial G^k} = \frac{\beta' F_K k + \beta \beta' F_{KK} k}{-\beta^2 F_{KK}} = \frac{\beta' F_K k}{\alpha \beta' F_L l} \sigma - \frac{\beta'}{\beta} k,
\]

(10)

\[
\frac{\partial k}{\partial G^l} = \frac{\alpha' F_{KL} l}{-\beta F_{KK}} = \frac{\alpha'}{\alpha} k.
\]

(11)

where \( \sigma \) is the elasticity of substitution between \( l \) and \( k \): \( \sigma = F_L F_K/F_{LK} F \). The second equality is based on the nature of constant returns to scale technology: \( F_{LK} \alpha l + F_{KK} \beta k = 0 \); \( F_{LK} > 0 \), \( F_{KK} < 0 \). As for (9), it is straightforward to see that the negative impact of the tax on \( k \) is correlated to \( \sigma \). Equation (10) shows that an increase in \( G^k \) affects the demand for the mobile factor in two different manners. The first term on the RHS captures the impact on the productivity of \( k \) that increases the demand for the mobile factor. This productivity impact is related to the elasticity of substitution because it gives private firms an incentive to substitute the mobile factor for the immobile one by changing relative productivity of private factors. On the other hand, the second term of (10) shows that the rise in \( G^k \), by increasing the effective supply of the mobile factor, \( \beta k \), causes firms to reduce their demand for \( k \) due to decreasing marginal returns.\(^4\) Equation (11) implies that an increase in \( G^l \) increases the effective supply of the immobile factor and, thus, the

\(^4\) Because of this impact, \( \partial k/\partial G^k \) can be negative. This occurs when the immobile factor’s share of income, \( \alpha F_L / F \), exceeds \( \sigma \).
demand for the mobile factor is increased through complementarity between private factors. Note that the impacts of \( G_k \) and \( G_l \) on the effective factor supplies depend on the proportional change in \( \beta \) or \( \alpha \), respectively.

In this paper, the impacts of “balanced-budget” policy changes on the tax base are relevant to the analysis. For later use, the balanced-budget relations between policy variables are provided. Differentiating the public budget constraint, (3), and using (6)–(8) gives

\[
kd_t - \beta' F_K k d G^k - \alpha' F_L l d G^l = 0. 
\]

(12)

Any marginal policy change must satisfy (12) in equilibrium.

3.1. Levels of public input provision

This section considers the provision level of each factor-specific public input. The first-order condition for \( G_i \), (7) or (8), and (6) yield the following provision rules:

\[
\beta' F_K k - 1 = -t \left( \beta' F_K \frac{\partial k}{\partial t} + \frac{\partial k}{\partial G_k} \right), 
\]

(13)

\[
\alpha' F_L l - 1 = -\left( \frac{\alpha' F_L l}{k} \frac{\partial l}{\partial t} + k \frac{\partial l}{\partial G_l} \right), 
\]

(14)

From (12), the RHS of (13) and (14) is equal to \(-t[(\partial k/\partial t)(\partial t/\partial G^l) + \partial k/\partial G^l]\) where \( \partial t/\partial G^k = \beta' F_K \) and \( \partial t/\partial G^l = \alpha' F_L l / k \). If the increase in \( G^l \) financed by the mobile-factor tax reduces the tax base, \( G^l \) is underprovided in the sense that its marginal product exceeds the marginal cost.\(^5\) Applying (9)–(11) to (13) and (14) gives

\[
\beta' F_K k - 1 = tk \frac{\beta'}{\beta}, 
\]

(15)

\[
\alpha' F_L l - 1 = tk \frac{\alpha'}{\alpha} \left( \frac{F}{\beta F_K k} - 1 \right). 
\]

(16)

Equations (15) and (16) yield the following proposition:

**Proposition 1.** Tax competition leads to underprovision of \( G^k \). If the elasticity of substitution exceeds the mobile factor’s share of income, \( \beta F_K k / F, G^l \) is underprovided.

The standard tax competition argument concerning underprovision applies to public input complementing mobile factors. From (9) and (10), the RHS of (15) corresponds to the impact of \( G^k \) on the effective supply of \( k \), because the impact on the productivity of \( k \) is exactly offset by the impact of the balanced-budget tax increase on the tax base.

\(^5\) In the present model, it is easy to confirm that this argument is consistent with the welfare impact of coordinated policy changes, which has been frequently examined in the tax competition literature; see, for example, Keen and Marchand [7], Matsumoto [10] and Wilson [13]. Consider an economy with identical, competitive regions. (My analysis in the main text deals with the behavior of a representative region.) Starting from a symmetric equilibrium, a coordinated increase in \( G^l \) made by all regions improves welfare if and only if the marginal product exceeds the marginal cost in the equilibrium.
The sign of (16) is ambiguous depending on the relative magnitude of the impact of \( G^l \) and the impact of the tax increase. Still, public input complementing immobile factors will also be underprovided unless \( \sigma \) is considerably low.

### 3.2. The mix of factor-specific public inputs

I now turn to a consideration of the mix of factor-specific public inputs under tax competition. Equations (7) and (8) give

\[
\alpha' F_{Ll} - \beta' F_{Kk} = t \left( \alpha' F_{Ll} \frac{\partial k}{\partial G^k} - \beta' F_{Kk} \frac{\partial k}{\partial G^l} \right).
\]  

This equation describes the equilibrium condition for the expenditure mix in the present model. The LHS represents the difference in the marginal products between \( G^l \) and \( G^k \). In a first-best setting where a lump-sum tax is available, the mix of these public inputs would be set such that \( \alpha' F_{Ll} = \beta' F_{Kk} \). The RHS is the distortionary impact caused by the mobile-factor tax. The parenthesized term is closely related to the change in \( k \) when \( G^k \) is marginally increased while decreasing \( G^l \) to balance the public budget. If this policy change raises \( k \), then \( \alpha' F_{Ll} > \beta' F_{Kk} \), implying that the regional government spends too much on \( G^k \) and too little on \( G^l \) in order to expand the tax base.  

Formally, (12) implies that (17) can be rewritten as

\[
\alpha' F_{Ll} - \beta' F_{Kk} = t \frac{\alpha' \beta'}{\beta} F_{Kk} (\sigma - 1).
\]  

This equation gives the following proposition:

**Proposition 2.** If the elasticity of substitution is more than (less than) one, too much tax revenue is spent on \( G^k \) (\( G^l \)) and too little on \( G^k \) (\( G^l \)). If the elasticity is equal to one, the mix of these inputs is efficient relative to the equilibrium tax revenue.

Based on (10) and (11), the RHS of (19) can be decomposed into the impact on the productivity of the mobile factor and the impact on the effective factor supplies. The impact of \( G^k \) on the productivity of \( k \) corresponds to the term, \( (\alpha' / \alpha)(\beta' / \beta) F_{Kk} \). This impact generates a bias towards relative overprovision of \( G^k \). On the other hand, the rise in \( G^k \) increases the effective supply of the mobile factor while the balanced-budget decline in \( G^l \) decreases the effective supply of the immobile factor. These impacts, which are combined
to obtain the term, $-\left(\frac{\alpha'}{\alpha}\right)\left(\frac{\beta'}{\beta}\right)Fk$, create a bias towards relative underprovision of $G^k$. The size of $\sigma$ determines the relative magnitude of these two terms. In the special case where $\sigma = 1$, the mix of $G^l$ and $G^k$ is efficient even though these inputs are underprovided; see Proposition 1.\footnote{This result is akin to that of Matsumoto [11] where if the production function is CES and if labor and capital are mobile, the mix of public goods and public inputs is efficient under tax competition even though the overall expenditure level is inefficiently low.} Except for this case, tax competition gives rise to an inefficient mix of factor-specific public inputs. A higher value of the elasticity leads to an inefficiently high amount of $G^k$ relative to $G^l$. For example, if $k$ is regarded as capital while $l$ is labor, Proposition 2 shows that under $\sigma > 1$, tax competition gives regional governments an incentive to attach too much importance to providing infrastructure, in order to compete for business capital, rather than improving the quality of human capital in their territories. Alternatively, one may consider labor mobility. (In this case, the model assumes that the immobile factor is land and that labor can commute across regions but cannot migrate.) Then, $\sigma > 1$ means that too much tax revenue is spent on education and manpower training and too little on land development, in order to expand the labor tax base.

Lastly, note that the parameters representing the impact of public input provision on production technology, $\alpha'$ and $\beta'$, are not important to the nature of the equilibrium expenditure mix. This unimportance can be seen from (18). In this equation, the sign of the RHS is determined by the relative magnitude of $\partial k/\partial G^k$ and $(\partial k/\partial G^l)(\partial G^l/\partial G^k)$. Equations (11) and (12) imply that the impact of the balanced-budget decline in $G^l$ on the tax base depends on $\beta'$, rather than $\alpha'$, just like the impact of $G^k$ on $k$. Thus, the difference between $\alpha'$ and $\beta'$ is not crucial to the present qualitative analysis. (As a matter of course, one needs to know the value of these parameters in order to quantify the mix inefficiency of public input provision.)

4. Concluding remarks

Because the sub-national level of government provides various public inputs to encourage regional development, the composition of expenditures on productive activities, as well as the overall level of public input provision, is an important policy issue. To analyze this issue in the context of tax competition, this paper has introduced the concept of factor-specific public inputs. This concept is closely related to the familiar Harrod-neutral and Solow-neutral technical change. Although the problem of public inputs mix will be sensitive to how those inputs are defined and classified, this concept enables a very clear result to be derived from a very simple framework and, thus, the present analysis provides a helpful benchmark for future research of the composition of public budgets.

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