The Evolution of Income and Fertility Inequalities over the Course of Economic Development: A Human Capital Perspective

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Using an endogenous-growth, overlapping-generations framework in which human capital is the engine of growth, we trace the dynamic evolution of income and fertility distributions and their interdependencies over three endogenous phases of economic development. In our model, heterogeneous families determine fertility and children's human capital, and generations are linked via parental altruism and social interactions. We derive and test discriminating propositions concerning the dynamic behavior of inequalities in fertility, educational attainments, and three endogenous income inequality measures—family-income inequality, income-group inequality, and the Gini coefficient. In this context, we also reexamine the "Kuznets hypothesis" concerning the relation between income growth and inequality.

I. Introduction

Using historical data from a number of developing and developed countries, Kuznets (1955, 1963) argued that income inequality first rises and then falls during a transitional development period. Post-Kuznets studies fall into two main groups. The first retested his hypothesis against data from many other countries.[1] The second dealt with the development-

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[1] Some studies favor the Kuznets hypothesis: e.g., Lindert and Williamson (1985) and Barro (2000). Others reject it or find no systematic relation, e.g., Fields (1990), Anand and Kanbur (1993), Fields and Jakubson (1994), and Deininger and Squire (1998). Studies of the way income growth *rate* relates to income inequality also report mixed results: Alesina and Rodrik (1994), Persson and Tabellini (1994), and Deininger and Squire (1998) find a negative relation; Forbes (2000) finds a positive one; Barro (2000) finds that higher

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inequality nexus theoretically as a causal relation going either from growth to inequality or vice versa.[2] Both sets of studies offered conflicting conclusions about the competing theoretical and empirical hypotheses.

One issue that was hardly addressed so far is the relevance of relative fertility choices of different income groups for understanding the evolution of income inequalities over all phases of economic development, including the comparative levels of income inequality across less-developed, stagnant economies and highly developed, persistently growing ones. The relevance of fertility and related population trends to income distributions is twofold: measures of the "size distribution of income," such as the shares of specific income brackets in total income or the Gini coefficient, are weighted by the population shares of different income brackets, which reflect underlying fertility differences across these brackets. Further, even individual- or family-based income inequality measures are indirectly affected by parental choices concerning fertility and investment in human capital of offspring. This is especially important over the transitional development period, which typically involves a "demographic transition" as well.

We tackle this issue by developing an overlapping-generations endogenous-growth model with heterogeneous families, in which human capital is the engine of growth and parents determine fertility and educational investments in offspring. Our deterministic model offers a dynamic extension of Becker's (1967) model of income distribution, as well as a generalization of more recent work by Ehrlich and Lui (1991), Zhong (1998), and Ehrlich and Yuen (2000). We show that the behavior of income inequality over the transitional development phase can vary in different countries, depending on the factors triggering economic takeoff and their impact on fertility and investment behavior of different family groups.

Our model suggests that the relationship between income growth and income inequality is *associative*, not causal. Three main forces influence this association: (*a*) interactions between overlapping generations within families, (*b*) heterogeneities in endowments and investment efficiencies

inequality lowers the growth rate in poor countries but raises it in rich ones, while Banerjee and Duflo (2000) find an inverted-U relation between the two.

[2] Models supporting Kuznets's direction of causality rely, e.g., on structural shifts in a two-sector model (Kuznets 1955, 1963; Anand and Kanbur 1993); skill-biased technical progress (Eicher 1996; Aghion, Caroli, and Garcia-Penalosa 1999); and organizational changes (Kremer and Maskin 1996; Lindbeck and Snower 1997; Acemoglu 1999). Models favoring causality going from inequality to growth rely, e.g., on credit market imperfections (Loury 1981; Banerjee and Newman 1993; Galor and Zeira 1993; Durlauf 1996; Galor and Moav 2004); political economy changes (Venieris and Gupta 1986; Alesina and Perotti 1996; Benhabib and Rustichini 1996); and fertility changes by income (Kremer and Chen 2002; De la Croix and Doepke 2003).

across families, and (*c*) social interactions across families.[3] The model consists of finitely lived individuals. Parents optimize on investments in the quantity and quality of children (in an extended version outlined in App. A, on savings as well). Persistent heterogeneities and social interactions across families enable us to derive equilibrium paths of income, schooling, and fertility distributions over three phases: a stagnant steady state, characterizing economies in a pretakeoff stage; a perpetual-growth steady state, more related to economies in a highly developed stage; and a transition phase linking the two.

By this approach we are also able to provide new insights about the "Kuznets hypothesis." The association between income growth and inequality is influenced partly by the parameters determining the inequality levels at the growth versus stagnant steady state. The association is in a state of flux during the transitional development phase. Specifically, we show that over this phase the paths of alternative income inequality measures can assume a U shape, an inverted-U shape, or combinations of the two, with the inequality level ultimately falling, rising, or staying the same, depending on the way heterogeneity sources are correlated across families, how relative fertility and population shares are affected across groups, and the inequality measure used. We derive three such measures as endogenous variables: family-income inequality, income-group inequality, and the Gini coefficient. A distinct implication of our model is that regardless of the specific shape of the income inequality path over the transitional development phase, fertility inequality can be expected to assume an inverted-U shape, with both tails converging on virtual equality.

Section II introduces the model. In Section III we derive equilibrium regimes and comparative dynamic implications, and in Section IV we present simulated dynamic paths of our basic inequality measures. Section V presents supportive evidence on the dynamic evolution of fertility, schooling, and income inequality based on international panel data from 1950 to 1998.

II. The Model and Equilibrium Solutions

A. The Economic Environment

To derive income and fertility inequality paths over the entire development process, we extend the deterministic representative-family, over-

[3] Lucas (1988) also considers spillover effects in goods production, stemming from the average human capital level in a representative-agent model. Tamura (1991) applies a similar spillover effect in human capital production, which results in full income convergence. De la Croix and Doepke (2003) also use the Lucas-type spillover effect and consider fertility as well as income inequality but reach the same income-convergence result as Tamura. They focus on growth effects stemming from assumed initial inequality in human capital, however, rather than on the joint dynamic evolution of inequalities in education, income, and fertility over the entire development process. Zhong (1998) and Ehrlich and Yuen (2000) develop prototype frameworks similar to ours but ignore the role of fertility.

lapping generations model of endogenous growth in Ehrlich and Lui (1991) to a heterogeneous-family case that recognizes interfamily interactions as well.

1. The Economy

The economy comprises heterogeneous family groups of varying income levels. For simplicity we illustrate the model by recognizing just two family groups indexed by $i = 1, 2$: a leading (1) and a following group (2), which in principle could switch places over the process of development. We implicitly rely on positive assortative mating within groups to maintain their separate identity in any stable steady state, because if intergroup mating is allowed and children inherit the average characteristics of their parents, human capital attainments and income would eventually converge in all families, given our deterministic setup.[4] In the benchmark model agents live over two periods: childhood, $t-1$, when human capital is formed through parental investments, and parenthood, t (but see App. A). All family-based decisions are made by working parents.

Similar to Becker (1967), we focus on three objective sources of inherited heterogeneity: (*a*) differences in learning or production abilities (A^i); (*b*) differences in income-yielding "endowments" ($\overline{H}^i$), stemming from inherited social status, political power, or other personal assets; and (*c*) differences in education-financing costs (θ^i). We abstract from differences in preferences or external production technologies, since these need not be related to idiosyncratic personal differences.

2. Goods Production and Income

The economy is competitive, and human capital is the sole asset. Production or earning capacity of a parent in group i, $(\overline{H}^i + H_t^i)$, is com-

[4] Becker (1973) and Burdett and Coles (1997) offer evidence supporting positive assortative mating by intelligence and education. Our assumed fixed distribution of family types still allows for family-group mobility over time, since family group 2 may move closer to, or even overtake, group 1 over the transitional phase (see n. 14). Alternatively, we can allow for individual mobility as well by allowing for stochastic deviations in ability within initial k subgroups of family type i ($= 1, 2$), using a stochastic specification similar to that of Becker and Tomes (1979): $A_{t+1}^{k,i} = \phi A_t^{k,i} + (1-\phi)A^i + \varepsilon_{t+1}^{k,i}$, where $A_{t+1}^{k,i}$ is the ability level of generation $t+1$ in subgroup k, A^i is the mean ability level of group i, $0 < \phi < 1$ is a weight, and $\varepsilon_{t+1}^{k,i}$ is a stochastic variable, independently and identically distributed, with zero mean and constant variance. It is reasonable to assume that (*a*) $\varepsilon_{t+1}^{k,i}$ is revealed a posteriori, i.e., after parental decisions on quantity and quality of children are made; and (*b*) social interaction is determined by the average earning capacity and family group size in group 1 relative to 2 in generation t. Allowing for positive assortative mating within each subgroup, we now obtain regression toward the mean ability level A^i inside group i in any steady state. Overall income inequality in a steady state will be larger now than in our deterministic model due to the influence of $\varepsilon_{t+1}^{k,i}$. Inequalities in fertility and educational investments, however, will be the same as in the deterministic case, unless we also allow for a stochastic component distinguishing desired and realized fertility. The steady-state comparative-dynamic results of our model hold in this stochastic specification as well.

posed of a fixed, inherited income-producing endowment ($\overline{H}^{i}$) measured in units of human capital, and a stock of human capital, (H_t^i), attained through parental inputs. Labor supply is assumed fixed. For convenience, we also assume that consumer goods, including educational services, can be purchased. Under a linear and strongly additive production technology for all goods, aggregate earnings (Y) equals aggregate production capacities in each period (L), and firms' zero-profit condition, $\pi = Y - \varpi L = 0$, yields a time-invariant real rental rate per unit of production capacity, $\varpi = 1$, which also guarantees full employment. We initially ignore savings, so earnings are identical to income. In Appendix A, we allow for savings and thereby for an old-age period of life for retired parents and show that our inferences concerning earnings inequality hold for income inequality as well.

3. Human Capital Production

The human capital formation rule is given by

$$H_{t+1}^i = A^i h_t^i (\overline{H}^i + H_t^i)^{1-\gamma} \left[(\overline{H}^1 + H_t^1) \left(\frac{N_t^1}{N_t^i} \right) \right]^{\gamma} \equiv A^i h_t^i (\overline{H}^i + H_t^i)(S_t^i)^{\gamma}, \quad (1)$$

where h_t^i is the share of earning capacity ($\overline{H}^i + H_t^i$) a parent from family group i ($i = 1, 2$) invests in educating each child, $\overline{H}^i$ and H_t^i are the parent's endowed and attained human capital stocks, respectively, and N_t^1/N_t^i is the ratio of the population shares of parents in family groups 1 and i. The term S_t^i denotes an interfamily, "social interaction" factor, which we define below. Note that for the top income group $i = 1$, equation (1) becomes $H_{t+1}^1 = A^1 h_t^1 (\overline{H}^1 + H_t^1)$.

Equation (1) captures two types of interactions in human capital production within and across families: (*a*) persistent human capital formation can be sustained over time only if parents invest in their children's knowledge; and (*b*) knowledge attained by agents with the higher earning capacity augments the attained production capacity of, and thus has a spillover effect on, agents with lower capacity (see below). Human capital formation is thus a social, as well as a private, process.

The intergenerational interaction is captured by the relationship between H_{t+1}^i and H_t^i in equation (1). The interfamily interaction is defined by the term $(S_t^i)^{\gamma}$ in equation (1), where $S_t^2 \equiv [(\overline{H}^1 + H_t^1)/(\overline{H}^2 + H_t^2)](N_t^1/N_t^2) \equiv E_t^2 P_t^2$. The ratios E_t^2 and P_t^2 reflect the relative earning capacities and family-group sizes of agents in group 1 relative to 2 in generation t, and $\gamma < 1$ is a spillover-intensity parameter. This specification is designed to capture social interaction as a knowledge spillover effect by which agents with lower earning capacity, or effective knowledge (group 2), benefit from interactions with leaders in knowledge (group 1) in various contexts.[5]

The relevance of E_t^2 is straightforward: the greater the disparity in knowledge, the greater the potential learning benefit to members of group 2 from knowledge transfer from members of group 1, given their own knowledge. The expression $P_t^2 \equiv N_t^1/N_t^2$ captures the determinant of effective interaction between individual members of groups 1 and 2, which is simply the conventional "teacher-student ratio."[6]

4. Preferences and Motivating Forces

For the sake of parsimony, we take parental altruism to be the sole force motivating parents' demand for children. The utility function of agent i at period t is

$$U(C_{1,t}^i, W_{t+1}^i) = \left(\frac{1}{1-\sigma}\right)[(C_{1,t}^i)^{1-\sigma} - 1] + \delta\left(\frac{1}{1-\sigma}\right)[(W_{t+1}^i)^{1-\sigma} - 1], \quad (2)$$

where δ denotes the inverse intertemporal discount factor, and σ the inverse intertemporal elasticity of substitution in consumption. In equation (2), $C_{1,t}^i$ denotes consumption of parents:

$$C_{1,t}^i = (\overline{H}^i + H_t^i)(1 - v^i n_t^i - \theta^i h_t^i n_t^i). \quad (3)$$

The variable n_t^i represents the number of children per parent, treated as a continuous and certain variable. The endogenous size of group i thus evolves over time as $N_{t+1}^i = N_t^i n_t^i$. The parameters v^i and θ^i are fixed unit costs, as fractions of earning capacity, of raising a child and financing educational investments per child, respectively. The latter may vary significantly across family groups because of capital market imperfections. The last term in equation (2),

$$W_{t+1}^i \equiv B(n_t^i)^\beta (H_{t+1}^i)^\alpha, \quad \text{with } \alpha = 1 \text{ and } \beta > 1, \quad (4)$$

represents a parental altruism function in an overlapping generations context, borrowed from Ehrlich and Lui (1991), which reflects psychic

[5] Knowledge transmission is thus modeled as an exogenous process, assuming that spillover effects cannot be internalized a priori. We also disallow strategic group behavior aimed at benefiting from spillover effects.

[6] The leader-follower specification can be generalized under additional assumptions to a J-group specification ($i = 1, \ldots J$): let the homogeneous members of group 1 be the exclusive source of knowledge transfer and assume that successful knowledge transfer from a member of group 1 to a member of group $i > 1$ requires *pairwise interactions*, such as random pairings of agents forming a work team and sharing a school desk or a two-seat assignment on a commuter plane, which allow for *intensive* knowledge transfer. The odds that the member of $i > 1$ is paired with a member of any other group $j \neq i$, is $(N - N^i)/N^i$, where N is the total population. Effective interaction requires, however, that the paired member of group j would be a member of group 1, the conditional probability of which is $N^1/(N - N^i)$. The odds of exclusive interaction between members from groups 1 and i is thus the product of the two $[(N - N^i)/N^i][N^1/(N - N^i)] = P^i = N^1/N^i$. This restrictive specification of social interaction yields all the propositions derived in this article.

rewards that parents obtain vicariously from children's number and attained human capital.[7] The restrictions on α and β are necessary to obtain interior solutions for both h_t^i and n_t^i. To ensure the concavity of equation (2) we must further restrict $\alpha(1-\sigma)<1$ (or $\sigma>0$) and $\beta(1-\sigma)<1$.

B. *Basic Solutions*

The objective function (2) is maximized by choosing n_t^i, and h_t^i, subject to (1), (3), and (4), taking $\{H_t^i, H_t^1, N_t^i, N_t^1\}$ as given. The first-order conditions for optimal n_t^i and h_t^i are

$$0 = -(C_{1,t}^i)^{-\sigma}(v^i + \theta^i h_t^i)(\overline{H}^i + H_t^i) + \delta(W_{t+1}^i)^{-\sigma}\beta B(n_t^i)^{\beta-1}(H_{t+1}^i)^{\alpha} \quad \text{for } n_t^i \geq 0 \tag{5}$$

and

$$0 = -(C_{1,t}^i)^{-\sigma}\theta^i n_t^i(\overline{H}^i + H_t^i) + \delta(W_{t+1}^i)^{-\sigma}\alpha B(n_t^i)^{\beta}(H_{t+1}^i)^{\alpha-1}A^i(\overline{H}^i + H_t^i)(S_t^i)^{\gamma} \quad \text{for } h_t^i \geq 0. \tag{6}$$

Equations (5) and (6) confirm that in order for interior solutions for n_t^i and h_t^i to coexist over all development phases, we must restrict $\beta>\alpha$ and $\alpha=1$ (note that a growth equilibrium cannot be sustained if $\alpha>1$). The optimal solutions for h_t^i and n_t^i are then found to be

$$h_t^i = \frac{v^i}{\theta^i(\beta-1)}, \quad \text{or} \quad \theta^i h_t^i = \frac{v^i}{\beta-1} < 1 \tag{7}$$

and

$$\left(1 - \frac{\beta v^i n_t^i}{\beta-1}\right)^{-\sigma} = \delta\left[B\left(\frac{A^i}{\theta^i}\right)(S_t^i)^{\gamma}\right]^{1-\sigma}\left(\frac{v^i}{\beta-1}\right)^{-\sigma}(n_t^i)^{\beta(1-\sigma)-1}. \tag{8}$$

Note that in equation (8), the left-hand and right-hand sides represent convex, monotonically rising and falling marginal cost and benefits schedules of n_t^i, respectively. Their intersection thus offers unique solutions for h_t^i and n_t^i (the latter having an upper limit of $n^i = [(\beta-1)/(\beta v^i)]$).

By equation (7), the equilibrium value of h_t^i is independent of the level of human capital, essentially because for altruistic parents a change in H raises proportionally both the marginal benefits and costs of investment H. An interesting feature of equation (8) is that ability, A^i, and unit financing cost of education, θ^i, exert opposite effects on n_t^i. In fact,

[7] This specification of altruism or "companionship," relating parents' utility to children's human capital rather than earning capacity, has the advantage of allowing for interior solutions for all our control and "state" variables, which permits the derivation of all inequality measures as endogenous variables all along the development process, including both the stagnant and growth steady states.

the solution depends on the ratio $e^i \equiv A^i/\theta^i$, or families' relative "investment efficiencies." Equation (8) indicates that equilibrium fertility, $n_t^i = n(\cdot) = n(v^i, B, \beta, e^i, \delta, S_t^i)$, falls with v^i and rises with the other parameters entering $n(\cdot)$.

C. *Income Inequality Measures*

Three income inequality indices become endogenous functions of our model's state variables.

a. $E_t^2 \equiv (\overline{H}^1 + H_t^1)/(\overline{H}^2 + H_t^2)$ is a *family-income inequality* index: the ratio of the (full) incomes of individual families in family group 1 relative to family group 2. An inequality measure directly related to E_t^2 in our model is inequality in attained human capital stocks, H_t^1/H_t^2, which may be captured by the standard deviation of schooling attainments.

b. $S_t^2 \equiv [(\overline{H}^1 + H_t^1)/(\overline{H}^2 + H_t^2)](N_t^1/N_t^2) \equiv E_t^2 P_t^2$ is our *income-group or income-bracket inequality* index—a product of relative income levels and group sizes of family group 1 relative to 2— which is also a component of the social interaction term in equation (1). It measures the fraction of aggregate income going to the top income bracket (above a given dollar value), relative to the lower bracket. Note that $P_t^2 = N_t^1/N_t^2$ is a related inequality measure—an *income-group-size inequality* index. It measures the proportion of families in the top income bracket relative to those in the lower bracket, or the relative population shares of families in the two income brackets. The latter is not independent of S_t^2 and E_t^2 since, by definition, $P_t^2 \equiv S_t^2/E_t^2$.

c. The *Gini coefficient*, $G_t \equiv (S_t^2 - P_t^2)/[(1 + S_t^2)(1 + P_t^2)]$, turns out to be a nonlinear function of S_t^2 and P_t^2. Specifically, it is an increasing function of S_t^2, but a decreasing function of P_t^2.

An immediate insight from all of these income inequality measures is their inherent dependence on the population shares of different family groups, resulting from their fertility choices.

III. Equilibrium Regimes and Comparative Dynamics

Equations (7) and (8) represent a recursive model, since the leading group 1 arrives at all of its choices independently as a function of its own parameters, while the following group 2's fertility choices are affected by those of family 1 through the social interaction term, $(S_t^2)^\gamma$. From equations (1) and (7) we derive an explicit, linear law of motion of human capital in group 1:

$$H_{t+1}^{1} = \left[\frac{v^{1}(A^{1}/\theta^{1})}{\beta - 1}\right]H_{t}^{1} + \left[\frac{v^{1}(A^{1}/\theta^{1})}{\beta - 1}\right]\overline{H}^{1}. \tag{9}$$

Since the economy is dictated by family 1, the equation of motion (9) indicates the existence of two equilibrium regimes, depending on the magnitudes of the model's basic parameters: If the slope $dH_{t+1}^{1}/dH_{t}^{1} = v^{1}(A^{1}/\theta^{1})/(\beta - 1) = A^{1}h_{t}^{1}$ exceeds one, H_{t}^{1} grows exponentially without bound and the economy is in a persistent growth equilibrium regime. If the slope is below one, H_{t}^{1} becomes constant and a stagnant-equilibrium regime ensues. The transitional development phase connecting the regimes' steady states is supported by the same parameter set that sustains the growth regime.

For equilibrium steady states to exist, however, certain outcomes must hold.

Proposition 1. Both fertility rates and marginal rates of change of human capital stocks in different family groups must converge at any stable equilibrium steady state. Formally, we can show that

$$n_{t}^{1} = n_{t}^{2} \quad \text{and}$$

$$a_{t}^{1} \equiv \left(\frac{dH_{t+1}^{1}}{dH_{t}^{1}}\right) = A^{1}h_{t}^{1} = a_{t}^{2} \equiv \left(\frac{dH_{t+1}^{2}}{dH_{t}^{2}}\right) = A^{2}h_{t}^{2}(S_{t}^{2})^{\gamma}. \tag{10}$$

Proof. In any equilibrium steady state that preserves the heterogeneous family groups, their relative population shares, $P_{t}^{2} \equiv N_{t}^{1}/N_{t}^{2}$, must be constant over time, requiring fertility rates to equalize across families.[8] Suppose there is an exogenous shock that initially lowers n^{2} below n^{1}. This will increase P_{t}^{2} and the social interaction term S_{t}^{2} in equation (8), raising the marginal benefit from children in family 2 but not in family 1, which is unaffected by S_{t}^{2}. The rise in n_{t}^{2} subsequently depresses P_{t}^{2} and S_{t}^{2}, and these adjustments continue until desired fertility rates equalize (although not necessarily actual rates if the latter were subject to purely stochastic deviations from desired fertility rates; see n. 4). This result is consistent with optimal fertility choices, since in a stable steady state the impact of lower income is offset by a proportionally lower shadow price of fertility.[9] Similarly, in a balanced growth steady state, H_{t}^{1}/H_{t}^{2}, for example, must be constant over time. This requires the

[8] If we allow for differences in survival probabilities from childhood to adulthood, π^{i}, the necessary stability condition would be equality of the expected numbers of surviving children: $\pi^{1}n^{1} = \pi^{2}n^{2}$.

[9] As propositions 2 and 3 below indicate, in any stable equilibrium steady state, all families spend the same proportion of their potential income on quantity (vn) and quality (θh) of children. Thus all wind up with the same desired fertility level despite their different income levels, because in equilibrium, a lower income ($Y_{t}^{i} = \overline{H}^{i} + H_{t}^{i}$) would be offset by a proportionately lower shadow price of fertility ($[v^{i} + \theta^{i}h_{t}^{i}]Y_{t}^{i}$), and the demand for children's quantity n^{i} is then a function of the ratio of the two.

growth rate of human capital, which is the same as the latter's marginal rate of change or total derivative (dH^i_{t+1}/dH^i_t) at the growth steady state, to equalize across groups. This result will be shown to apply in a stagnant steady state as well (see Sec. III.*A*).

Given our assumed uniformity of preferences and external production parameters, $M \equiv \{B, \beta, \gamma, \delta, \sigma\}$, dynamic stability also requires a parameter restriction: the unit cost of raising a child as a fraction of earning capacity must be identical across families, or $v^1 = v^2$. Put differently, only initial endowments $(\overline{H}^i)$, abilities (A^i), and unit investment-financing costs (θ^i) are allowed to vary across families. This *heterogeneity restriction* necessarily holds in the log utility case.[10] More generally, suppose that $v^1 > v^2$. Equation (7) implies that the income share invested in educating a child would then be higher in family 1 relative to 2, $\theta^1 h^1_t > \theta^2 h^2_t$. By proposition 1 the rate of growth of human capital must be identical across families in a stable growth steady state, $A^1h^1 = A^2h^2(S^2_t)^\gamma$. These equations imply that $(A^1/\theta^2) < (A^2/\theta^2)(S^2_t)^\gamma$; that is, family 2's marginal cost of fertility schedule would locate below that of family 1, while its marginal benefit schedule would locate above family 1's. Optimal fertility would then be strictly higher in family 2 relative to 1, negating a stable equilibrium. To assure consistent parameter restrictions we must have $v^1 = v^2 = v$ in all development phases.

A. Stagnant Equilibrium Steady State (s)

In a stagnant equilibrium (SE) steady state, the control and state variables and all inequality measures are constant over time, given the parameters affecting the rate of return on human capital, A^i/θ^i and v. If the latter are sufficiently low, so $(dH^1_{t+1}/dH^1_t) = A^1h^1_t = v(A^1/\theta^1)/(\beta - 1) < 1$ in equation (9), equation (1) for agent 1 would necessarily converge on SE from any arbitrary value of H^1_0. Local stability conditions for agent 1 require, in addition, that $dn^1_t/dN^1_t < 1$. For agent 2, and thus the full system, the necessary and sufficient conditions for local stability can be shown to require that the elasticity of fertility $n^2(s)$ with respect to the social interaction term $S^2(s)$ would lie within the following range: $0 < e(s) < 2\{1 - [\gamma K/(K + 1)]\}$, where $e(s)$ is the elasticity of $n^2(s)$ with respect to the social interaction term $S^2(s)$, or $e(s) \equiv \partial \ln [n^2(s)]/\partial \ln [S^2(s)]$, and $K \equiv \{H^2(s)/[\overline{H}^2 + H^2(s)]\} < 1$. In our numerical simulations using the baseline parameters of table 1, part 1, this condition is satisfied for any permissible values of the social interaction intensity $0 \leq \gamma \leq 1$ (see App. B.1).

Proposition 2. In a stable SE steady state, family-income inequality, $E^2(s)$, and families' relative human capital attainments equal their rel-

[10] In the log utility case, we have an explicit solution for fertility: $n^i_t = [\delta(\beta - 1)]/[v^i(1 + \delta\beta)]$. Thus, for n^1 to be equal to n^2, $v^1 = v^2$.

ative inherited income endowments:

$$E^2(s) = \left(\frac{H_t^1}{H_t^2}\right)(s) = \frac{\overline{H}^1}{\overline{H}^2}. \tag{11}$$

Proof. Equation (11) is obtained utilizing equation (1), the stagnancy of human capital attainments in the SE, and the expectation that $a^1(s) \equiv A^1h^1(s) = a^2(s) \equiv A^2h^2(s)[S^2(s)]^\gamma$ (proposition 1). Indeed, the latter must hold in the SE as well as the growth equilibrium (GE): by the heterogeneity condition $v^1 = v^2$, equation (7) implies that

$$\theta^1 h^1(s) = \theta^2 h^2(s). \tag{12}$$

Inserting the condition $a^1(s) = a^2(s)$ in equation (8) is thus seen as necessary to guarantee that $n^1(s) = n^2(s)$.

Proposition 2 implies that status differences are the key factor determining family-income inequality in economies that are stagnant over long periods, an inference that seems compatible with historical evidence, such as pre–Industrial Revolution Europe. Equation (11) is also dynamically stable: suppose we start from a stable SE. If a parameter shock lowers a^2, raising H_t^1/H_t^2 above $\overline{H}^1/\overline{H}^2$, then E_t^2 and S_t^2 would also rise initially. This would raise n_t^2 and depress P_t^2 and S_t^2, which in turn would increase a_t^2 and lower H_t^1/H_t^2 until the initial equilibrium is restored.

Note that by equations (10) and (12), the income shares spent on both raising (vn) and educating (θh) children equalize across families. Combining these with equation (8) allows us to derive the SE value of our income-group inequality measure:

$$S^2(s) \equiv E^2(s)P^2(s) = \left(\frac{A^1/\theta^1}{A^2/\theta^2}\right)^{(1/\gamma)} \equiv \left(\frac{e^1}{e^2}\right)^{(1/\gamma)}. \tag{13}$$

Proposition 3. In a stagnant steady state the shares of earning capacity devoted to human capital investments per child, $\theta^i h^i(s)$ are equalized across all family groups, and income-group inequality, $S^2(s)$ depends strictly on the relative "investment efficiencies" of family 1 relative to 2, $(e^1/e^2)^{(1/\gamma)}$. Unlike $S^2(s)$, however, the size distribution of families across income brackets $P^2(s)$ and the Gini coefficient $G(s)$ depend on both relative family endowments and investment efficiencies.

Equation (13) has an intuitive interpretation: the only way the social interaction term $S^2(s) \equiv E^2(s)P^2(s)$ can adjust to satisfy equation (10) is through adjustments in fertility choices and their impact on $P^2(s)$. Fertility choices are strictly a function of relative investment efficiencies (eq. [8]). Therefore, adjustments in $S^2(s)$ must be a function of relative investment efficiencies as well.

TABLE 1

SIMULATING COMPARATIVE DYNAMIC EFFECTS OF PARAMETER CHANGES IN A TWO-AGENT ECONOMY

Part 1. Stagnant Equilibrium

A^1/θ^1	A^2/θ^2	$\overline{H}^1$	$v^1(v^2)$	γ	$B^1(B^2)$	$n^1(n^2)$	$Y^1 = \overline{H}^1 + H^1$	$Y^2 = \overline{H}^2 + H^2$	E	S	$P = N^1/N^2$	Gini
2/1	1/1.01	50	.01	.4	.1	8.053	55.555	1.111	50	5.799	.116	.749
3/1	1/1.01	50	.01	.4	.1	8.243	58.824	1.176	50	15.981	.320	.699
3/1	**1.5/1.01**	50	.01	.4	.1	8.243	58.824	1.176	50	5.799	.116	.749
2/1	**1.5/1.01**	50	.01	.4	.1	8.053	55.555	1.111	50	2.105	.042	.637
2/1	1/1.01	**60**	.01	.4	.1	8.053	66.666	1.111	60	5.799	.097	.765
2/1	1/1.01	50	**.015**	.4	.1	5.344	58.824	1.176	50	5.799	.116	.749
2/1	1/1.01	50	.01	**.45**	.1	8.053	55.555	1.111	50	4.770	.095	.740
2/1	1/1.01	50	.01	.4	**.15**	8.243	55.555	1.111	50	5.799	.116	.749

Part 2. Growth Equilibrium

A^1/θ^1	A^2/θ^2	$v^1(v^2)$	γ	$B^1(B^2)$	$n^1(n^2)$	h^1	h^2	$a^1 = A^1h^1$	E	S	$P = N^1/N^2$	Gini
40/1	20/1.01	.01	.4	.1	9.448	.05	.0495	2	50	5.799	.116	.749
50/1	20/1.01	.01	.4	.1	9.551	.05	.0495	2.5	85.138	10.131	.119	.804
50/1	**25/1.01**	.01	.4	.1	9.551	.05	.0495	2.5	50	5.799	.116	.749
40/1	**25/1.01**	.01	.4	.1	9.448	.05	.0495	2	29.368	3.320	.113	.667
40/1	20/1.01	**.015**	.4	.1	6.274	.075	.0743	3	50	5.799	.116	.749
40/1	20/1.01	.01	**.45**	.1	9.448	.05	.0495	2	41.502	4.770	.115	.724
40/1	20/1.01	.01	.4	**.15**	9.634	.05	.0495	2	50	5.799	.116	.749

Part 3. Takeoff Triggers

	A^1	A^2	θ^1	θ^2	$v^1(v^2)$	$n^1(n^2)$	h^1	h^2	E	S	$P = N^1/N^2$	Gini
SE	2	1	1	1.01	.01	8.053	.05	.0495	50	5.799	.116	.749
GE	**40**	**20**	1	1.01	.01	9.448	.05	.0495	50	5.799	.116	.749
GE	**40**	1	1	1.01	.01	9.448	.05	.0495	38,643	10,374	.268	.788
GE	2	1	**1/15**	**1.01/15**	.01	9.316	.75	.743	50	5.799	.116	.749
GE	2	1	**1/15**	**1/15**	.01	9.316	.75	.743	48.857	5.657	.116	.746
GE	2	1	1	1.01	**.11**	.712	.55	.545	50	5.799	.116	.749

Note.—Parameters values that deviate from our benchmark values are presented in bold print. Part 1: Comparative dynamics in the stagnant steady state are simulated by changing A^1/θ^1, A^2/θ^2, $\overline{H}^1$, $v^1(= v^2)$, γ, or $B^1(= B^2)$, holding constant all other parameters: $\overline{H}^2 = 1$, $\sigma = .9$, $\delta = .9$, and $\beta = 1.2$. Columns for h^1 and h^2 are suppressed, but the corresponding solutions are .05, .0495 for all rows except row 6; and .075, .0743 for row 6. Part 2: Comparative dynamics in the growth steady state are simulated by changing A^1/θ^1, A^2/θ^2, $v^1(= v^2)$, γ, or $B^1(= B^2)$, holding constant $\sigma = .9$, $\delta = .9$, and $\beta = 1.2$. Part 3: Simulations show the impact of uniform (proportionate) and nonuniform changes in A^i and θ^i, as well as in the common level of $v^1 = v^i$ that generate a takeoff from the SE to GE steady state, holding constant $\overline{H}^1 = 50$, $\overline{H}^2 = 1$, $\sigma = .9$, $\delta = .9$, $\gamma = .4$, $\beta = 1.2$, and $B^1 = B^2 = .1$.

This analysis yields a set of comparative-dynamic implications in the SE steady state. By proposition 2, family-income inequality, $E^2(s)$, is strictly a function of families' relative endowments. By proposition 3, any changes in relative income-group inequality, $S^2(s) \equiv E^2(s)P^2(s)$, thus occur via the groups' relative population shares, $P^2(s) = (N^1/N^2)(s)$. For example, an increase in group 1's relative investment efficiency e^1/e^2 would increase $S^2(s)$ by increasing its population share, $P^2(s)$. A higher intensity of knowledge spillover, γ, leaves family choices intact but lowers $P^2(s)$, hence, $S^2(s)$. Changes in other common parameters can affect $n^i(s)$ and $h^i(s)$ but not any income inequality measures: by equations (7) and (8), higher fertility unit costs (v) increase h^i and lower n^i, while stronger altruistic preferences (B) raise n^i in all families (see table 1, part 1).

Does income inequality change systematically at different income levels? Note that while income levels are stagnant in an SE, they can vary with parameter shifts: a skill-biased technological advance raising family 1's relative investment efficiency, e^1/e^2, ultimately raises income levels in all families. By proposition 2, however, family-income inequality, $E^2(s)$ remains unchanged, while by proposition 3, income-group inequality, $S^2(s)$, rises because the wealth effect generated by a higher e^1 temporarily increases group 1's relative fertility and, ultimately, its population share $P^2(s)$. The impact on Gini is ambiguous, since $G(s)$ rises with $S^2(s)$ but falls with $P^2(s)$.[11] The association between income levels and inequality at the SE thus depends on the inequality measure used.

B. *Growth Equilibrium Steady State* (g)

In a steady state of a balanced GE, human capital stocks H_t^i grow exponentially without bound, while the long-run values of $n^i(g)$, $h^i(g)$, and all inequality measures are constant. Since the relative impact of the endowments, $\overline{H}^i$, vanishes, proposition 1 implies that the long-run growth rate of human capital in all groups converges on its marginal value in family group 1, $\lim_{t\to\infty}(H_{t+1}^i/H_t^i) \equiv a^1(g) = A^1h^1(g) = a^i(g) = A^ih^i(g)S^i(g)^\gamma$. Local stability for group 1 is assured if $a^1(g) > 1$, or $v(A^1/\theta^1) > (\beta - 1)$, since $dh_t^1/dH_t^1 = 0$ and $dn_t^1/dN_t^1 \le 0$ as in the SE. The necessary and sufficient conditions for the local stability of the full system, which guaranties a unique solution for income-group inequality, $S^2(g) = E^2(g)P^2(g)$ but not for its component parts (see the discussion below), are shown in Appendix B.2 to require that $\{-\gamma < e(g) < 2 - \gamma\}$,

[11] More specifically, $\partial G/\partial x = (1 - 1/E^2)(1 - P^2S^2)\partial S^2/\partial x + S^2(P^2 + 1/E^2)^2\partial E^2/\partial x$ in any equilibrium position, where x is one of our parameters. In a stagnant state, a rise in (e^1/e^2) unambiguously raises $S^2(s)$, while not affecting $E^2(s)$. Thus $G(s)$ will rise or fall depending on whether $P^2(s) \cdot S^2(s)$ is smaller or bigger than one. An increase in $\overline{H}^1/\overline{H}^2$ will not affect $S^2(s)$ but will raise unambiguously $E^2(s)$ and $G(s)$. In a growth steady state, we cannot make symmetrical predictions because an increase in (e^1/e^2), e.g., may also affect $E^2(g)$, as our analysis in Sec. III.*B* indicates.

where $e(g)$ is the elasticity of $n^2(g)$ with respect to $S^2(g)$. Again, our numerical illustration using the baseline parameters in table 1, part 2, indicates that this condition is satisfied for all permissible values of $0 \leq \gamma \leq 1$ (see App. B.2).

Proposition 4. Proposition 3 remains valid at the growth equilibrium steady state as well. Moreover, if the relative distribution of the heterogeneous parameters A^i and θ^i remains the same in the SE and GE, income-group inequality S^2 would converge on the same level in both steady states:

$$S^2(g) \equiv E^2(g)P^2(g) = \left(\frac{A^1/\theta^1}{A^2/\theta^2}\right)^{(1/\gamma)} \equiv \left(\frac{e^1}{e^2}\right)^{(1/\gamma)} = S^2(s). \qquad (14)$$

The proof is the same as for proposition 3. The comparative-dynamic implications of equation (14) are also similar to those of equation (13): $S^2(g)$ rises with the relative investment efficiency (e^1/e^2) and falls with the spillover coefficient γ, as is the case in the SE steady state. No unique solutions exist, however, for the GE values of $E^2(g)$ or $P^2(g)$ (see App. B.2). Unlike the stagnant steady-state case, where family-income inequality $E^2(s)$ was determined strictly by the relative family endowments, the endowments' influence vanishes with persistent growth. The comparative values of E^2 in the GE versus SE steady states thus depend on the evolution of E_t^2 along the transitional phase. The same holds for the income-group-size inequality index, P_t^2.

Comparative-dynamics effects of parameter shifts on inequalities in family income, $E^2(g)$, and income-group size, $P^2(g)$, are thus ambiguous. A *skill-biased* technological or institutional advance favoring the leading family, which by proposition 4 unambiguously raises $S^2(g) \equiv E^2(g)P^2(g)$, must raise either $E^2(g)$ or $P^2(g)$ or both. The value $E^2(g)$ necessarily rises if the upward shift in A^1 raises the growth rate of human capital and income in family 1, $A^1h_t^1$, above that in family 2 over the entire transitional adjustment path. As in the SE case, fertility rises as well, as shown by our simulations in table 1, part 2, because an increase in A^1 generates a wealth effect, which favors fertility over educational investment. Over the transitional dynamics adjustment, family 1's higher income growth rate ultimately lifts the income growth rate and fertility in family 2 as well, due to the rising social interaction effect $(S_t^2)^\gamma$. Thus, unlike the SE case, both family-income inequality $E^2(g)$ and income-group-size inequality $P^2(g)$ rise while fertility inequality increases over the initial part of the adjustment period but ultimately falls and vanishes at a new GE steady state. The change in Gini is ambiguous, since G rises with S^2 and falls with P^2, but our simulations indicate that $G(g)$ rises as well, as $S^2(g)$ rises more sharply.

These results appear to be consistent with the U.S. experience following the "information technology revolution": empirical studies have shown that the Gini coefficient rose in the 1980s (e.g., Katz and Murphy

1992; Acemoglu 2002). As we predict, census data also show that over the same period total fertility levels reversed a historic downward trend since the baby boom and started rising from 1.74 in 1977 to a peak of 2.08 in 1990, remaining stable thereafter at about 2.02, while the coefficient of variation of fertility rose from 0.619 in 1983 to 0.710 in 1994, but with the rate of increase diminishing since then (see fig. 1).[12]

Table 1, part 2, also confirms the implications of equations (7) and (8) that a rise in v lowers $n^i(g)$ and raises $h^i(g)$, thus the growth rate, while a rise in B raises only $n^i(g)$. Neither affects any income inequality measure. This is in contrast to a skill-biased technical advance, which raises all income inequality measures and the growth rate as well. The association between income growth rate and income inequality thus depends on the parameter changes responsible for their comovements.

C. *Takeoff Triggers*

As equation (9) predicts, starting at a stagnant equilibrium, an upward shock in investment efficiency, A^i/θ^i, or in the common unit cost of raising children, v, even one affecting just group 1, can generate a takeoff and a transitional development phase for all groups. Under the altruistic specification of our model, the "demographic transition" typically accompanying a takeoff, whereby fertility generally declines, can be generated by a sufficient upward shift in the unit cost share of raising children, v, but not by a technological advance: the latter generates a higher growth rate and thus a wealth effect favoring fertility (see table 1, part 3).[13] By propositions 3 and 4, however, shifts in v, thus in fertility *levels*, have no bearing on the dynamic evolution of our income inequality measures over the transitional development phase. Also, the shapes of the evolution paths of all our income inequality measures, that is, whether income inequality is monotonically rising or falling or has a U shape, or an inverted U shape, or any combinations therein, is dictated just by shifts in the parameters affecting investment efficiency (A^i and θ^i). The paths of income inequality that we derive below are

[12] Although the fertility level of Hispanic female immigrants is higher than that of the U.S. female population, the rise in the U.S. total fertility rate during the 1980s cannot be entirely attributed to the rise in the share of Hispanic immigrants in the female population (ages 15–49) from 2.37 percent to 4.63 percent in the 1980s. As U.S. Census data show, although 93 percent of Hispanic immigrants report themselves as whites, the rise in the total fertility rate among whites is approximately the same as that of nonwhites, and the total fertility rate of blacks also significantly rose from 2.17 percent in 1980 to 2.48 percent in 1990.

[13] Major technological advances raising investment efficiency typically generate structural shifts in the economy favoring employment opportunities for women as well as a rise in housing costs due to urbanization and are thus accompanied by an increase in the unit (opportunity) costs of bearing and raising children (v). Technological advances can thus generate a demographic transition indirectly by our preset model. In Ehrlich and Lui (1991), where parents are motivated by old-age support from educated children as well as by pure altruism, technological advances can generate a demographic transition directly.

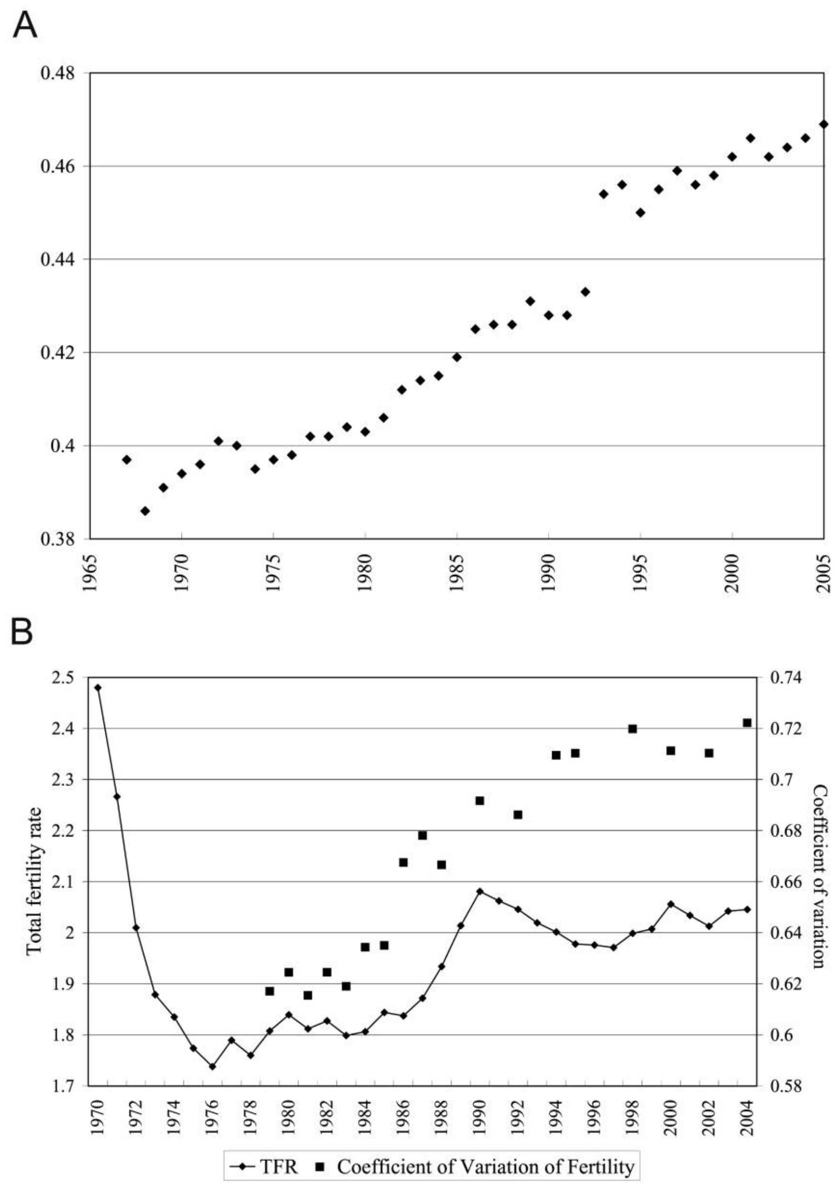

Figure 1.—Changes in the dynamic patterns of the Gini coefficient, total fertility rate (TFR), and coefficient of variation of the fertility distribution in the United States in recent decades. *A*, Gini coefficient. *B*, Total fertility rate and coefficient of variation of the fertility distribution. *B* shows the coefficient of variation of the distribution of surviving children per woman age 40–44.

thus independent of the evolution of fertility levels but are affected by the evolution of relative fertility levels across groups and their relative population shares, P_t^2, as our analysis in the following section demonstrates.

IV. Inequality Paths over the Transitional Development Phase

A. *Paths of Income Inequality Measures*

The preceding analysis indicates that the behavior of inequalities over the development phase partly depends on the type of shock that produces a takeoff. An equally important issue is how fast any given shock reaches different family groups: a skill-biased technological advance, for example, is likely to first reach the group with the highest investment efficiency or affect it proportionally more than others. While this group need not necessarily be the one with the highest income—this depends on the correlation between ability and initial endowments across family groups—a positive correlation is likely (see Becker 1967). To contain the possible scenarios we focus on three that are neither exhaustive nor necessarily of equal empirical plausibility.

1. Synchronous and Uniform Shocks

These shocks affect all takeoff-triggering parameters (A^i/θ^i, v) simultaneously and by the same proportion. This case can be dubbed "the neutral equilibrium path"; we can show that over the transitional phase

$$S_t^2 = S^2(s) = S^2(g) = \left(\frac{A^1/\theta^1}{A^2/\theta^2}\right)^{(1/\gamma)} \quad \text{and}$$

$$E_t^2 = E^2(s) = E^2(g) = \frac{\overline{H}^1}{\overline{H}^2}. \tag{15}$$

Put differently, our basic earnings inequality measures chart a horizontal path all along the development process. This is because a uniform proportional increase in a takeoff-triggering parameter affects all optimality conditions symmetrically, leaving constant the spillover effect. Since the Gini coefficient is a function of S^2 and P^2, it also exhibits a flat transition path.

2. Shocks Favorable to Family 1

Such a shock affects family 1 proportionally more than, or ahead of, family 2. An example would be a technical advance or a market reform that enhances just the productivity of especially skilled workers (A^1), such as a shift from a command to a market economy, or one that ultimately enhances the productivity of all agents proportionally but is

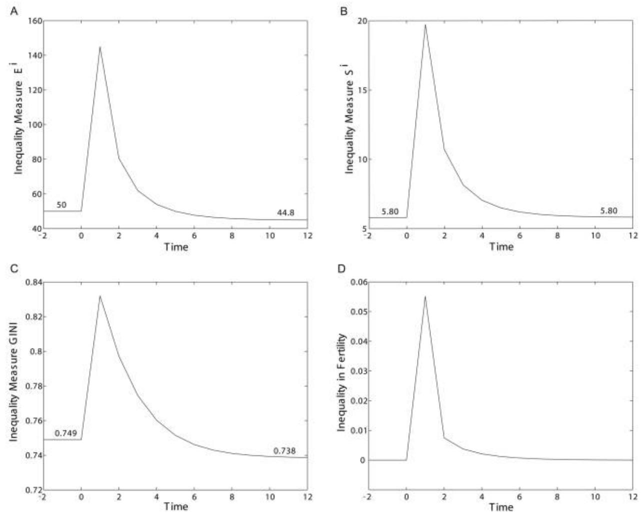

Figure 2.—Simulated time paths of the evolution of key endogenous variables over the process of development: a uniform productivity shock affecting family 1 ahead of family 2. *A*, Family-income inequality (E). *B*, Income-group inequality (S). *C*, Gini coefficient (G). *D*, Inequality in fertility. Parameter values used in these simulations are $\theta^1 = 1$, $\theta^2 = 1.01$, $H_0^1 = 50$, $H_0^2 = 1$, $B^1 = B^2 = 0.1$, $v^1 = v^2 = 0.01$, $\gamma = 0.4$, $\sigma = 0.9$, $\delta = 0.9$, and $\beta = 1.2$. Prior to period 1, the economy is in a stable SE steady state, with $A^1 = 2$ and $A^2 = 1$. In period 1, family 1 alone experiences a once-and-for-all increase in A^1 to 40. In period 2, family 2 also experiences the same proportional increase in A^2 to 20.

first integrated by family 1, such as the information technology revolution. We implicitly assume a positive correlation between income-generating endowments and efficiency at human capital investments, or Cov $(\overline{H}^i, A^i/\theta^i) > 0$, so the higher-income family 1 is a leading group at both the stagnant and growth steady states.

If family 1 is affected ahead of family 2, the transitional development phase would be characterized by the coexistence of family groups in different stages of transition: family 1 would initially become a "growth family" while the other remains a "stagnant family." But the persistent growth in family 1's income ultimately produces a takeoff for all, and by proposition 1 all will eventually grow at an equal rate. The time paths of all income inequality measures (S^2, E^2, and G) will exhibit an inverted-U shape, consistent with the "Kuznets hypothesis" (see fig. 2).

The shapes of the income inequality paths in all scenarios, including the comparative income inequality levels in the growth, relative to the stagnant steady states, depend strictly on whether the technology shift ultimately affects all families "uniformly," that is, equiproportionally, or nonuniformly. If a skill-biased technology shift ultimately becomes uniform, it does not affect the GE income-class inequality $S^2 \equiv E^2P^2$ by equation (14), but it lowers the GE family-income inequality $E^2(g)$, because the wealth effect triggered by the jump in A^1 initially raises the

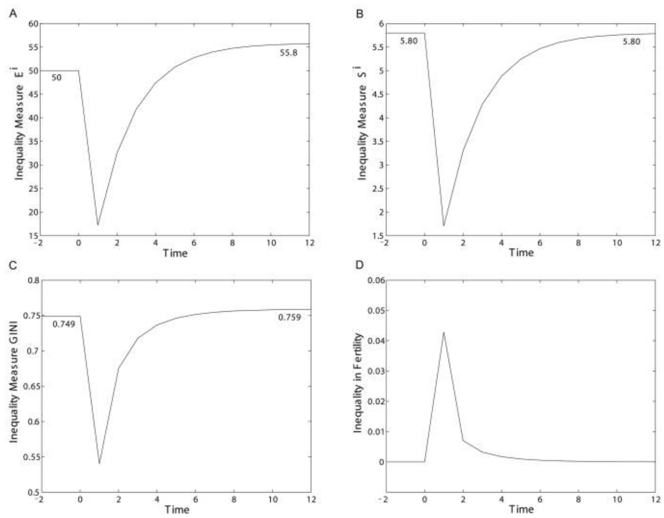

Figure 3.—Simulated time paths of the evolution of key endogenous variables over the process of development: a uniform productivity shock affecting family 2 ahead of family 1. *A*, Family-income inequality (E). *B*, Income-group inequality (S). *C*, Gini coefficient (G). *D*, Inequality in fertility. Parameter values used in the simulations: $A^1 = 2$, $A^2 = 1$, $H_0^1 = 50$, $H_0^2 = 1$, $B^1 = B^2 = 0.1$, $v^1 = v^2 = 0.01$, $\gamma = 0.4$, $\sigma = 0.9$, $\delta = 0.9$, and $\beta = 1.2$. Prior to period 1, the economy is in a stable SE steady state with $\theta^1 = 1$ and $\theta^2 = 1.01$. In period 1, family 2 alone experiences a once-and-for-all reduction in θ^2 to 1.01/20. In period 2, also family 1 experiences the same proportional decrease in θ^1 to 1/20.

relative fertility level of group 1 and, ultimately, its relative population share, $P^2(g)$. If the shock raises A^1 proportionally more for family 1, income inequality would then be monotonically increasing over the development phase for all our three income inequality measures.

3. Shocks Favorable to Family 2

A family 2 friendly shock could occur, for example, when a less segmented capital market lowers the education financing cost to all families, but especially to family 2, thus lowering (e^1/e^2), or when the shock first benefits family 2, which could not initially finance private schooling. In this case, the takeoff-triggering shocks will produce transition paths just opposite to those in case *b*. The time paths of all inequality measures will assume a U shape if family 2 experiences a takeoff shock ahead of family 1 (see fig. 3). Whether the inequality *level* rises or falls at the GE relative to the SE steady state depends on whether the nonsynchronized shock ultimately becomes equiproportional, in which case the income-bracket inequality, S^2, is constant and $P^2(g)$ falls, so family-income inequality, $E^2(g)$, rises. If investment efficiency rises proportionally more for family 2, all income inequality levels would be monotonically decreasing, or family 2 may overtake family 1.[14] Our simulations of cases *b* and *c* also reveal negative associations between income growth rate and income inequality over the transitional development phase. In case *b*, income inequality and per capita income growth rate are negatively associated at an early stage of the transition but become positively associated at a more advanced stage, as Barro (2000) finds, while in case *c* they are negatively associated, which is what Forbes (2000) finds. Our analysis thus shows that the dynamic association between income growth and income inequality can vary by the specific takeoff triggers or at different stages of the transitional development phase.

Regardless of the way a takeoff-generating shock affects different families, a fundamental implication of our model is that the shape of the family-income inequality path over the transitional development period, $E_t^2 \equiv (\overline{H}^1 + H_t^1)/(\overline{H}^2 + H_t^2)$, would always be congruent to that of human capital attainments, H_t^1/H_t^2, regardless of the specific shape of the paths. Our simulations indicate that this result applies to our other measures of income inequality as well.

B. *Paths of Inequality in Fertility and Human Capital Investment*

Since by proposition 1 desired fertility is equalized across families in the SE and GE steady states, while generally deviating across families over the transitional phase linking the two, we have the following proposition.

Proposition 5. Except in the "neutral equilibrium" case, the inequality path of desired fertility will exhibit an inverted-U shape but tend toward equality in the two steady states framing the transitional phase (see figs. 2*D* and 3*D*). In the neutral equilibrium case, inequality in desired fertility assumes a flat time path.

If income inequality measures assume an inverted-U shape due to a skill-oriented technology advance, as in case *b* of the preceding section, family 1's relative fertility will initially rise above that of family 2 and fall below it in subsequent periods, but ultimately result in a higher population share of group 1, $P^2(g)$. This pattern of evolution in relative fertility and income inequality would not be altered at all if the technology shock is followed by a rise in the cost of housing or opportunity costs of a female's time, which raises the unit cost share of bearing and raising children, v, and generates a demographic transition for both

[14] There is also the possibility of mixed cases. For example, a technological shock reaches family 1 first (case *b*), but government subsidization of education targets family 2 (case *c*). Alternatively, if a reduction in θ^2 affects family 2 many periods ahead of family 1, or by a sufficiently greater proportion, so that e^1/e^2 actually falls, family 2 can overtake family 1 and become the "leading family" in terms of income-generating capacity. Income inequality will then reach a minimum at the point of overtaking but will rise afterward until it converges on its GE steady-state level. In this case the time path of income inequality will assume an S shape.

families. This association between fertility rankings and income inequality is consistent with the findings in Kremer and Chen (2002) and De la Croix and Doepke (2003). If the income inequality path assumes a U shape as a result of lower financing costs benefiting lower-income groups, as in case *c* of the preceding section, family 1's fertility will initially fall short of, but then exceed, that of family 2 during the transition phase. Again, this shape of the income inequality path would not be altered if a subsequent shock in v generates a demographic transition with family 2's fertility level falling ahead of 1's. In our GE framework, however, such associations do not indicate causality nor can they persist, as desired fertility differences ultimately vanish in a steady state.

V. Empirical Analysis

A. *Basic Tests*

We test empirically two basic implications of the model: (*a*) by proposition 5, we expect fertility inequality to display an inverted-U shape with flat tails over the development phase; (*b*) since schooling levels may approximate human capital attainments, we can test another basic prediction: we expect the shapes of schooling and family-income inequality paths to be similar over the transition phase.[15] By our simulation analysis, this expectation applies to all our income inequality measures as well. To test these propositions we use panel data from different countries over periods of varying length. Since all countries in our sample exhibit positive growth rates over the period, they represent economies in transition toward a steady state of growth.

B. *Data and Variables Used*

1. *Completed fertility.*—Data on the distribution of surviving children per woman are available from the World Fertility Surveys and the Demographic and Health Surveys. Our sample is based on 72 surveys of 29 developing countries in various years between 1974 and 2000. From the individual-level data in each survey, we derive the distribution of surviving children of women age 40 and over. We make this restriction to ensure that our measures relate to women who completed childbearing. We then use the standard deviation of the distribution of surviving children per woman age 40 and over (SD-FERT) as our fertility inequality measure. But since the (SD-FERT) is subject to a secular drift, we enter the average level of completed fertility as a control variable, (AV-FERT).

[15] Our family-income inequality measure, E^2, converges on a steady-state level, $(H^1/H^2)(g)$, at the growth-equilibrium steady state. De Gregorio and Lee (2002) provide independent support. They estimate a positive relationship between inequality in educational attainments and income inequality.

2. *Human capital.*—Our data are taken from Barro and Lee (2000).[16] We use the average number of years of schooling in the population age 15 and over as a proxy for human capital stock. As a measure of inequality in educational attainments we use the standard deviation of the distribution of schooling years (SD-SCHYR) in the population age 15 and over. As in the fertility inequality regressions, we also add the mean schooling years as a control variable (AV-SCHYR).

3. *Income inequality.*—The data are taken from Dollar and Kraay (2001). These data cover 86 countries over the period 1950–98. No data are available about income-group relative inequality, S^i. We proxy our family-income relative inequality measure, E^i, however, by an interquintile income inequality ratio (QUINT; each "quintile" representing, by definition, an equal number of households), and Gini (G) by the conventional Gini coefficient (GINI). To be consistent with our model, we use only observations that are calculated from household income data, excluding observations based on personal income and expenditure data.

4. *Regressors.*—We use real per capita GDP level (RGDPn), reported in Heston, Summers, and Aten (2001), as a measure of the economy's development level. Note that since RGDPn is an endogenous variable in our model, its level summarizes the impact of the model's basic parameters that affect our dependent variables as well. We use government's share of GDP (GOV) as an additional regressor, however, as a proxy for government redistributional policies that may have an exogenous effect on our inequality measures. As a robustness check, we also enter the time trend as (T) to account for other possible missing trended indicators of the development phase. Summary statistics for all variables are given in Appendix C.

C. Regression Models

Our basic regression specification links our inequality measures as dependent variables with regressors introduced in the preceding paragraph plus AV-FERT and AV-SCHYR in the fertility and schooling inequality regressions, all in linear form, but RGDPn is entered in cubic or higher-order polynomial forms. This is because we predict the flattening of the income inequality, educational attainments, and fertility paths as the economy converges on a growth steady state.

Our regression specification is not intended to identify a causal effect going from RGDPn to the inequality measures, as these variables are simultaneously determined by our model. However, the use of RGDPn as an indicator of an economy's stage of development is likely to expose the estimated regression coefficients, which we use to depict the shape

[16] The Barro-Lee study reports average schooling years for four schooling levels in the population age 15 and up (zero, primary, secondary, and higher) and their population shares. We calculate the mean and standard deviation of this distribution for each country in all sample years.

of the inequality paths over the development process, to simultaneity biases. To derive unbiased and consistent estimated regression coefficients, we use a 2SLS (two-stage least squares) method in which RGDPn is treated as an endogenous variable. As instrumental variables (IVs) we include in the first-stage regression the inflation rate in log form, ln(INFLA), and one-year-lagged RGDPn. INFLA is used to capture the impact of inflation on the real economy, which is shown to affect RGDPn adversely in the first-stage regression. Alternatively, we also estimated the structural 2SLS regression equation using two-year-lagged or three-year-lagged RGDPn, as well as all three lagged RGDPn variables jointly as IVs, which produced very similar coefficient estimates. Basmann's test indicates that INFLA and each of the lagged RGDPn variables can serve as an IV in the first-stage regression, and when introduced as additional regressors in the second stage, each of these variables had insignificant and inconsistent effects, validating their use as IVs.

As robustness tests, models 1–3 in tables 2–5 present ordinary least squares regression results based on a few model modifications. In model 1 we enter only the basic regressors accounting for behavior of the relative inequality measures over the development phase. Model 2 allows for fixed country effects, which capture just within-country variability in all variables, and in model 3 we add also GOV and T as regressors. Models 4–6 in tables 2–5 repeat the specification of models 1–3 using the 2SLS method, which we use to derive figure 4. In the fertility regressions of table 2, we employ country-specific random-effects, instead of fixed-effects, models to increase the regressions' degrees of freedom, because the number of observations per country is small (2.6 on average). To test for serial correlation, we applied an AR(1) specification using models 2 and 5 of each table. The Durbin-Watson test cannot reject the null hypothesis of no autocorrelation in all cases.

D. Results

The fertility results are reported in table 2. All models in the table produce an inverted-U-shaped association between fertility inequality and real income. The estimated regression line we chose to depict in figure 4*A* is based on the 2SLS results from model 4 (with no random effects), because this allows for regression estimates based on variability in regressors both within and across the countries in our relatively small sample. The shape of the inequality path remains virtually the same, however, if we base it on model 5 (with random effects). Note that the sample is dominated by developing countries (in fig. 4*A*, fertility inequality peaks at an RGDPn level of $3,354, which means that 67 percent of the observations lie below this real GDP level). Therefore, if we extrapolate the regression line to RGDPn typical of developed economies, fertility inequality would drop sharply, as we predict theoretically. As for the effects of other regressors, the standard deviation of the

TABLE 2

Fertility Inequality Regressions

Dependent Variable: SD_FERT

	Model 1 OLS	Model 2 OLS with Random Effects	Model 3 OLS with Random Effects	Model 4 2SLS	Model 5 2SLS with Random Effects	Model 6 2SLS with Random Effects
Intercept*	1.776852	2.087804	2.181626	1.733183	1.985153	2.157891
	7.45	10.54	10.53	7.34	9.81	10.34
RGDPn	.000410	.000249	.000270	.000397	.000310	.000288
	2.23	1.59	1.83	2.55	2.19	2.21
$RGDPn^2$	−8.49E-08	−5.75E-08	−5.59E-08	−8.56E-08	−7.27E-08	−6.17E-08
	−1.90	−1.60	−1.66	−2.29	−2.33	−2.14
$RGDPn^3$	5.08E-12	3.40E-12	3.25E-12	5.25E-12	4.34E-12	3.64E-12
	1.64	1.44	1.47	2.10	2.24	2.02
AV_FERT	.056786	.042325	.134551	.071700	.051122	.146717
	1.88	2.14	4.18	2.38	2.71	4.90
GOV			−.004215			−.005876
			−.87			−1.28
T			−.013374			−.013659
			−3.46			−3.78
Adjusted R^2	.0854	.1637	.2824	.1004	.2340	.3852
Observations	72	72	72	70	70	70

Source.—Data sources are the World Fertility Surveys and the Demographic and Health Surveys (various years), the Office of Population Research, Princeton University.

Note.—The dependent variable is the standard deviation of the distribution of surviving children per woman age 40 and over, but we also add the mean fertility levels as a control variable (see text). Rows show the estimated coefficients (β) and their z-statistics (β/S_β). This table's regressions employ a random effects specification to account for missing idiosyncratic variables, because the number of observations per country is small. The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in models 2 and 5. The 2SLS model accounts for endogeneity of RGDPn. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

* The intercept coefficients represent the mean values of all intercept terms.

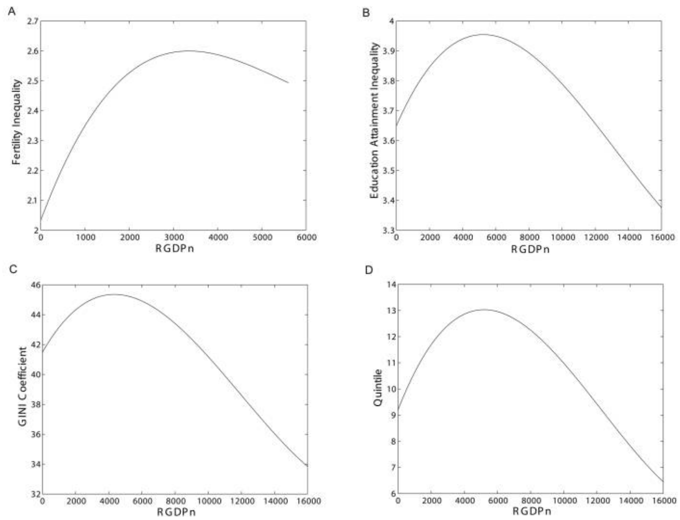

Figure 4.—Fitted regression lines linking the evolution of key inequality measures and per capita income levels within and across countries, based on 2SLS panel regression results. *A*, Mean-adjusted fertility inequality (SD-FERT). *B*, Mean-adjusted educational attainment inequality (SD-SCHYR). *C*, Gini coefficient (GINI). *D*, Interquintile ratio (QUINT). Panels *A*, *B*, *C*, and *D* are based on the regression results of model 4 in tables 2–5, respectively. The RGDPn values on the *X*-axes of all panels cover 90 percent of the observations on RGDPn used in our regressions.

fertility distribution is monotonically related to the distribution's mean, as one would expect for any distribution. GOV has a negative but insignificant estimated coefficient in the fertility regression (it has, however, a significant effect in the Gini regressions). The time-trend regressor is related inversely to fertility inequality but directly to educational attainments inequality.

Table 3 reports the results concerning inequality in educational attainments. Regression results without fixed effects indicate an inverted-U-shaped association between income and educational-attainment inequality, as depicted in figure 4*B* that is based on model 4. Results with fixed effects show a slightly different pattern (U shape at lower RGDPn values and inverted-U shape at higher values), essentially because schooling attainments have much lower variability among developed countries: the association between income and educational-attainment inequality is literally the same with or without allowance for fixed effects when only non-OECD country data are used. Mean schooling expectedly raises the SD of schooling.

Tables 4 and 5 present the regression results concerning the income inequality paths of *G* (Gini) and QUINT (a proxy for E_t^i), respectively. All model specifications indicate an inverted-U-shaped association between income inequality and income level. In models 4–6 of both tables we derive this association based on the subset of countries for which

TABLE 3

EDUCATIONAL ATTAINMENTS INEQUALITY REGRESSIONS

Dependent Variable: SD_SCHYR

	Model 1 OLS	Model 2 OLS with Fixed Effects	Model 3 OLS with Fixed Effects	Model 4 2SLS	Model 5 2SLS with Fixed Effects	Model 6 2SLS with Fixed Effects
Intercept*	2.634408	2.074857*	2.512515*	2.720585	2.227975*	2.560042*
	37.43			34.86		
RGDPn	9.29E-05	−3.79E-05	−4.36E-05	1.27E-04	−7.17E-06	−1.10E-06
	2.71	−1.02	−1.32	3.60	−.14	−.02
RGDPn^2	−1.20E-08	4.29E-09	2.75E-09	−1.51E-08	2.68E-09	2.35E-11
	−3.79	1.48	1.07	−4.74	.79	.01
RGDPn^3	3.03E-13	−1.59E-13	−1.19E-13	3.81E-13	−1.21E-13	−5.98E-14
	3.48	−2.21	−1.87	4.42	−1.52	−.85
AV_SCHYR	.205215	.351542	.111584	.189881	.302893	.051998
	11.97	18.40	4.50	10.26	11.64	1.79
GOV			.000617			.002328
			.25			.60
T			.029080			.032278
			12.92			12.38
Adjusted R^2	.3320	.4891	.6022	.3170	.4262	.5746
Observations	721	721	721	575	575	571

Source.—The data source is Barro and Lee (2000).

Note.—The dependent variable is the standard deviation in the distribution of schooling years attained in the population age 15 and over, but we also add the mean fertility levels as a control variable (see text). Rows show the estimated coefficients (β) and their z-statistics (β/S_β). Data on the dependent variable are available every five years. The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in models 2 and 5. The 2SLS model accounts for endogeneity of RGDPn. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

* The intercept coefficients represent the mean values of all intercept terms.

TABLE 4
INCOME INEQUALITY REGRESSIONS: GINI COEFFICIENT (G)
Dependent Variable: GINI

	Model 1 OLS	Model 2 OLS with Fixed Effects	Model 3 OLS with Fixed Effects	Model 4 2SLS	Model 5 2SLS with Fixed Effects	Model 6 2SLS with Fixed Effects
Intercept*	41.948920	35.729530*	39.392580*	41.489030	34.189310*	41.346450*
	26.34			24.10		
RGDPn	.001583	.001095	.001081	.001925	.001367	.000853
	2.94	2.62	2.03	3.45	2.37	1.17
RGDPn^2	−2.45E-07	−1.08E-07	−1.04E-07	−2.71E-07	−1.20E-07	−8.36E-08
	−5.22	−3.54	−3.00	−5.75	−3.03	−1.88
RGDPn^3	7.04E-12	2.81E-12	2.70E-12	7.55E-12	2.93E-12	2.10E-12
	5.96	3.99	3.43	6.51	3.38	2.18
GOV			−.170141			−.297255
			−2.00			−2.48
T			−.027090			.003112
			−.66			.05
Adjusted R^2	.4108	.0691	.0932	.4891	.0578	.0882
Observations	318	318	310	263	263	263

Source.—The data source is Dollar and Kraay (2001).
Note.—The dependent variable is the GINI coefficient, based on household income data. Rows show the estimated coefficients (β) and their z-statistics (β/S_β). The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in models 2 and 5. We derive the 2SLS regressions in models 4–6, which account for endogeneity of RGDPn, from the subset of countries for which both income and educational attainments data are available, in order to see if income and educational inequality paths exhibit a similar shape. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.
* The intercept coefficients represent the mean values of all intercept terms.

TABLE 5

INCOME INEQUALITY REGRESSIONS: INTERQUINTILE RATIO (E)

Dependent Variable: QUINT

	Model 1 OLS	Model 2 OLS with Fixed Effects	Model 3 OLS with Fixed Effects	Model 4 2SLS	Model 5 2SLS with Fixed Effects	Model 6 2SLS with Fixed Effects
Intercept*	9.110258	4.513832*	2.290842*	9.194144	7.860770*	9.954935*
	7.52			6.35		
RGDPn	.001440	.001294	.001478	.001622	.000626	.000485
	3.47	3.82	3.37	3.40	1.47	.87
RGDPn^2	−1.80E-07	−1.00E-07	−1.15E-07	−1.98E-07	−6.36E-08	−5.25E-08
	−4.92	−3.98	−3.95	−4.87	−2.14	−1.53
RGDPn^3	4.97E-12	2.32E-12	2.69E-12	5.37E-12	1.66E-12	1.41E-12
	5.34	3.96	4.02	5.34	2.53	1.87
GOV			.103520			−.086037
			1.50			−.95
T			−.005237			−.005231
			−.16			−.12
Adjusted R^2	.2498	.0663	.0788	.2908	.0447	.0505
Observations	289	289	281	240	240	240

Source.—The data source is Dollar and Kraay (2001), and only household income data are used.

Note.—The dependent variable is the share of total income received by the top, relative to the bottom, quintile of families in the population. Rows show the estimated coefficients (β) and their z-statistics (β/S_β). The Durbin-Watson test cannot reject the null hypothesis of no serial correlation in models 2 and 5. We derive the 2SLS regressions in models 4–6, which account for endogeneity of RGDPn, from the subset of countries for which both income and educational attainments data are available, in order to see if income and educational inequality paths exhibit a similar shape. Instrumental variables include, in addition to exogenous structural regressors, ln(INFLA) and one-year-lagged RGDPn.

* The intercept coefficients represent the mean values of all intercept terms.

both income and educational-attainments data are available. We do so because our model and simulation results imply that inequality paths of family-income (E_t^i) and human capital attainments would exhibit an increasingly similar shape as the economy converges on a GE steady state. This prediction is borne out in figure 4*C* and 4*D*, based on model 4 in each table.[17] In fact, the simple correlation coefficient of the predicted values shown in figure 4*B* and 4*D* is .999, and the correlation coefficient of indicator variables of the predicted paths in these figures (indicator variable takes the value of one if the slope of the path at each RGDPn value is nonnegative and zero otherwise) is .992. This lends support to our human capital approach to income distribution. In both figures, income inequality at the highest RGDPn level in our sample is lower than the extrapolated value of income inequality at RGDPn = 0, which is consistent with the path derived in case *a* of our simulations, whereby the income inequality measures *E* and GINI are lower at the growth steady state than at the stagnant steady state.

Since the estimated regression lines linking both educational attainments and income inequality measures with income levels exhibit an inverted-U shape, the results militate in favor of the Kuznets hypothesis. Note, however, that these results cannot be taken to support the Kuznets hypothesis as a general "law": our analysis indicates that the observed association can be affected by the specific composition of countries in our sample and their development stage, as well as by the specific takeoff triggers operating in different countries.

Our results concerning the dynamic behavior of income inequality can be compared to those of Deininger and Squire (1998). Although Deininger and Squire use the same data, they employ a different fixed-effects regression format with RGDPn and 1/RGDPn entered as regressors. When we add a cubic or higher-order form of RGDPn to the Deininger and Squire specification, however, the plotted relationships between GINI or QUINT and RGDPn exhibit inverted-U shapes in this specification as well, similar to those depicted in figure 4*C* and 4*D*.

VI. Concluding Remarks

Our deterministic model offers two main messages. The first is that income distribution in the population is linked fundamentally to the corresponding distribution of human capital attainments, not just under static conditions, as in Becker's 1967 paper, but under dynamic conditions as well. Contrary to inferences reached in earlier dynamic models (see n. 3), neither inequality disappears at an advanced level of development, let alone at a low and stagnant level of development—a conclusion justified by the observed systematic linkage between inequalities

[17] Also, regression results obtained when using a polynomial of RGDPn of the fourth, fifth, and sixth order showed the same pattern as in all panels of fig. 4.

in income and educational attainments even in highly developed economies. In this context, our propositions concerning the behavior of income inequality over the entire development process follow from the diverging trends in educational attainments across different income groups as well as the social interaction forces that bind them. The second, and more novel, message is that the dynamic evolution of the level and distribution of income over the entire development process cannot be fully understood without recognizing their linkage to the evolution of the relative fertility rates and population shares of different income classes. Our model shows how income and fertility distributions evolve conjointly over the transitional development phase. It also enables us to derive inferences about the comparative income inequality levels in the growth, relative to the stagnant equilibrium steady states, which frame the transition phase.

Regardless of the dynamic pattern of any of our income inequality measures, a distinct implication of our model is that the time path of fertility inequality will generally exhibit an inverted-U shape with attenuated tails over the transitional development phase. This prediction is supported by our empirical investigation, based on a sample of countries in different stages of development. It is also consistent with historical evidence indicating that the association between relative variances in fertility and income levels across Western European regions exhibited an inverted-U shape between the mid-nineteenth century and 1970, with fertility variances being quite low in the pre–demographic transition phase (see Coale and Treadway 1986).

Concerning the shape of the dynamic association between income growth and inequality, no "general law" applies. We offer, however, several new insights. First, the shape can vary depending on the parameter(s) that trigger the takeoff and the manner in which they reach different family groups. In ex-command economies, for example, where the opening of markets benefits more those with greater ability to acquire new knowledge, it is likely that case *b* illustrated in figure 2 emerges, with income inequality measures (initially) rising. If the takeoff trigger reflects largely government subsidization of education, case *c* illustrated in figure 3 may be more likely to occur. Second, the shape depends on whether the economy is in a stagnant or growth steady state, or in a transitional development phase where it becomes sensitive to the specific mix of countries in the sample. This may partly explain why different studies reach different conclusions about it. Third, the shape partly depends on the inequality measure used. We derive three such measures as endogenous variables: family-income inequality (E_t^2); income-group inequality ($S_t^2 = E_t^2 P_t^2$), which also depends on the distribution of families across income brackets (P_t^2); and the Gini coefficient (G_t), which is increasing in S_t^2 but decreasing in P_t^2. Our model offers some strong predictions concerning the dynamic behavior of these measures.

For example, under given preferences and external production technologies, and a stable distribution of investment efficiencies, we expect the income-group inequality measure, $S_t^2 = E_t^2 P_t^2$, to converge on equal levels in stagnant and growth steady states, or $S^2(s) = S^2(g)$. The comparative levels of this measure's components, E_t^2 and P_t^2, and therefore the Gini coefficient, in contrast, may vary across the two steady states. We expect family-income inequality in a stagnant steady state, $E^2(s)$, to be strictly a function of inequality in inherited family-specific endowments such as social or legal status. No clear-cut predictions can be made, however, about the shape of the family-income inequality path (E_t^2) over the transition phase: an inverted-U-shaped path is likely to emerge as a result of a uniform, skill-biased technological advance, which first reaches the top-skilled family group. The relative population share of that group (P^2) would then rise at the growth, compared to the stagnant, steady state, and in this scenario, family-income inequality would ultimately fall at the highest development level, compared to the lowest, as figure 4*D* illustrates. No government redistribution policies are required to achieve such outcome. In contrast, a U-shaped family-income inequality path with P^2 falling and family-income inequality turning higher at advanced, relative to pretakeoff, development phases, can emerge as a result of educational subsidies that lower the financing-cost disadvantage of especially lower-income families.

The link between income and fertility choices also sheds light on the behavior of income and fertility distributions at the stagnant- and growth-equilibrium steady states. In the SE, a skill-biased technology advance raises the steady-state income and fertility levels as well as fertility inequality and two income inequality measures—$S^2(s)$ and $G(s)$. In the GE, a similar advance raises income growth and all inequality measures, as well as fertility level and inequality. This prediction is borne out by the evidence we presented for the United States following the information technology revolution in the 1980s.

Although our analysis is based on a deterministic model of heterogeneous family groups with inherited differences in endowments and investment efficiencies, it allows for a degree of social mobility, especially in the case where leadership in human capital formation can switch from the group with initially highest earning capacity to the initially followers' group over the transitional development phase. Moreover, our key implications hold if we allow also for stochastic variations in ability within groups (see n. 4). And although our basic model relates to inequality in earning capacity, the propositions we derive may apply to total income as well (see App. A).

A critical implication of our model is that the dynamic path of family-income inequality, regardless of its shape, should mirror that of educational attainments, both having flat tails. This is what we find empirically. This finding supports the basic premise of our model, that family-income growth and distribution are directly linked to human capital formation and distribution.

Appendix A

The Model with Savings

Although our model abstracts from capital markets, we can incorporate returns on savings as an outcome of "home production" in which old parents' human capital serves as an input and the yield is subject to diminishing returns. This is a natural assumption in the context of our closed-economy framework. The extension allows us to recognize inequalities in labor earnings as well as in total income, incorporating both earnings and property income.

Formally, we now assume that each agent lives through three periods: childhood, young adulthood, and old age. Total savings is defined by $K_t \equiv (\overline{H}^i + H_t^i)s_t^i$, where s_t^i is the fraction of productive capacity saved at adulthood, and K_t is assumed to fully depreciate within one generation. Income from savings is generated when old parents combine their accumulated assets, K_t, with their human capital inputs via the production function, $F = D(\overline{H}^i + H_t^i)^{1-\kappa}[(\overline{H}^i + H_t^i)s_t^i]^{\kappa}$, $0 < \kappa < 1$. The relevant objective is to maximize

$$U(C_{1,t}^i, C_{2,t+1}^i, W_{t+1}^i) = \left(\frac{1}{1-\sigma}\right)[(C_{1,t}^i)^{1-\sigma} - 1] + \delta\left(\frac{1}{1-\sigma}\right)\{[(C_{2,t+1}^i)^{1-\sigma} - 1] + [(W_{t+1}^i)^{1-\sigma} - 1]\}, \qquad (2')$$

where the consumption flows at adulthood and old age are given by

$$C_{1,t}^i = (\overline{H}^i + H_t^i)(1 - vn_t^i - \theta^i h_t^i n_t^i - s_t^i),$$

$$C_{2,t+1}^i = D(\overline{H}^i + H_t^i)^{1-\kappa}[(\overline{H}^i + H_t^i)s_t^i]^{\kappa},$$

and the altruism function W_{t+1}^i is defined as in equation (4).

The first-order optimality conditions for n_t^i and s_t^i are thus given by

$$0 = -(C_{1,t}^i)^{-\sigma}(v + \theta^i h_t^i)(\overline{H}^i + H_t^i) + \delta(W_{t+1}^i)^{-\sigma}\beta B(n_t^i)^{\beta-1}(H_{t+1}^i)^{\alpha} \quad \text{for } n_t^i \geq 0 \qquad (\text{A1})$$

and

$$0 = -(C_{1,t}^i)^{-\sigma}(\overline{H}^i + H_t^i) + \delta(C_{2,t+1}^i)^{-\sigma}\kappa D(\overline{H}^i + H_t^i)(s_t^i)^{\kappa-1} \quad \text{for } s_t^i \geq 0. \qquad (\text{A2})$$

Combining these two equations, we can show that n_t^i and s_t^i are inversely related as follows:

$$1 - \frac{\beta v n_t^i}{\beta - 1} = s_t^i + (\delta D^{1-\sigma}\kappa)^{-1/\sigma}(s_t^i)^{[1-(1-\sigma)\kappa]/\sigma}.$$

It is easy to show that the optimal solution for h_t^i remains the same as in equation (7).

We can now distinguish income inequality from earnings inequality. The measures of the pooled income of a family head—earnings as well as property income from savings—can be defined parallel to our earnings inequality measures in Section II.*C*. For example, TS_t^2 below corresponds to the ratio of total income-group inequality (wage earnings of adult parents plus nonwage income of old parents) of group 1 relative to group 2, and the same holds for family income inequality, TE_t^2, and the Gini coefficient, TG_t:

$$TS_t^2 \equiv \frac{N_t^1(\overline{H}^1 + H_t^1) + N_{t-1}^1 D(\overline{H}^1 + H_{t-1}^1)(s_{t-1}^1)^{\kappa}}{N_t^2(H^2 + H_t^2) + N_{t-1}^2 D(H^2 + H_{t-1}^2)(s_{t-1}^2)^{\kappa}},$$

$$TE_t^2 \equiv \frac{TS_t^2}{TP_t^2}; \quad TP_t^2 \equiv \frac{N_t^1 + N_{t-1}^1}{N_t^2 + N_{t-1}^2},$$

and

$$TG_t \equiv \frac{TS_t^2 - (N_t^1 + N_{t-1}^1)/(N_t^2 + N_{t-1}^2)}{(1 + TS_t^2)/[1 + (N_t^1 + N_{t-1}^1)/(N_t^2 + N_{t-1}^2)]}.$$

Under our heterogeneity restriction, we can show that in any steady state, the optimal savings rates (s^i) and shares of income spent on raising and educating a child, (vn^i) and ($\theta^i h^i$), are identical in all family groups, since the first-order optimality conditions governing these control variables become identical for all family groups in all stable steady states. Our total income inequality measures are therefore identical to the corresponding earnings inequality measure at both the SE and GE steady states. Moreover, we can show that the relative inequality in earnings and, hence, in total income in this extended model is the same as that derived in our benchmark model sans savings, as given by equations (11), (13), and (14). All our propositions in Sections III and IV are also maintained in this extended model, as are the qualitative results of the comparative dynamics reported in table 1 for both the SE and GE steady states. The time paths of the inequality measures derived in Section IV are also shown to have the same shape as in the extended model with savings.

Over the transitional development phase, however, the savings rate may differ across families. For example, when a takeoff occurs as a result of a skilled-bias technological advance reaching initially the higher-income family group 1, our income inequality measures assume an inverted-U shape, and the savings rate of family 1 initially falls below that of (stagnant) family 2. In the following stage, however, as family group 2 experiences a takeoff because of the social-interaction effects coming from family group 1, its savings rate falls below that of family 1. The aggregate savings rate then starts rising while income inequality is falling. The resulting association between income inequality and the aggregate savings rate does not indicate causality, however (as in Keynes 1920; Kaldor 1957).

What would be the effect of changes in D or κ on our income inequality measures? As long as these changes are common to all families, they will affect only the composition of family income, but not the total income inequality measures, as our simulations confirm.

Appendix B

Local Stability Conditions in the Steady States

1. The Stagnant Equilibrium

At the stagnant equilibrium, the dynamic system is given by

$$P_{t+1}^2 = \frac{n^1}{[n^2(S_t^2)]P_t^2}, \tag{B1}$$

$$H_{t+1}^2 = A^2 h^2 (\overline{H}^2 + H_t^2)(S_t^2)^{\gamma}. \tag{B2}$$

To check for local stability, we first fully differentiate equations (B1) and (B2). Following some algebraic manipulations and omitting the superscript 2, the linearized system becomes

$$\begin{bmatrix} \hat{H}_{t+1} \\ \hat{P}_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\gamma)K & \gamma \\ eK & 1-e \end{bmatrix} \begin{bmatrix} \hat{H}_t \\ \hat{P}_t \end{bmatrix} = M \begin{bmatrix} \hat{H}_t \\ \hat{P}_t \end{bmatrix}, \tag{B3}$$

where $\hat{X}_t = d\ln(X_t)$ denote percentage deviations from steady-state values of $X = H$ or P, $K \equiv \{H(s)/[\overline{H} + H(s)]\}$, and $e \equiv [\partial n(s)/\partial S(s)]/[n(s)/S(s)]$. The characteristic polynomial of M or $\det(M - XI)$ is then given by

$$F(X) = X^2 + [e - (1-\gamma)K - 1]X + [(1-e)(1-\gamma)K - \gamma eK].$$

The necessary and sufficient condition for general stability (covering both oscillatory and nonoscillatory equilibrium scenarios) requires the two roots of the characteristic equation to be smaller than one in modulus. These conditions are summarized by

$$0 < e < 2\left[1 - \left(\frac{\gamma K}{K+1}\right)\right]. \tag{B4}$$

2. *The Growth Equilibrium*

In the growth equilibrium, the relative effect of $\overline{H}^i$ is negligible since H_t^i is increasing without bound. Thus we can suppress $\overline{H}^i$ to examine the growth equilibrium dynamics. We can then express the system of growth equilibrium in terms of E_t^2 and P_t^2 as follows:

$$P_{t+1}^2 = \frac{n^1}{n^2(S_t^2)} P_t^2, \tag{B5}$$

$$E_{t+1}^2 = \frac{H_{t+1}^1}{H_{t+1}^2} = \frac{A^1 h^1 H_t^1}{A^2 h^2 H_t^2 (S_t^2)^{\gamma}} = \frac{A^1 h^1}{A^2 h^2} E_t^2 (S_t^2)^{-\gamma} = \frac{A^1 h^1}{A^2 h^2} (E_t^2)^{1-\gamma} (P_t^2)^{-\gamma}. \tag{B6}$$

Taking the total differentials of (B5) and (B6) around the growth steady state, we see that the linearized system can be shown equal to

$$\begin{bmatrix} \hat{P}_{t+1} \\ \hat{E}_{t+1} \end{bmatrix} = \begin{bmatrix} 1-e(g) & -e(g) \\ -\gamma & (1-\gamma) \end{bmatrix} \begin{bmatrix} \hat{P}_t \\ \hat{E}_t \end{bmatrix} = M \begin{bmatrix} \hat{P}_t \\ \hat{E}_t \end{bmatrix}, \tag{B7}$$

where $e(g) \equiv [\partial n^2(g)/\partial S^2(g)]/[n^2(g)/S^2(g)]$.

The characteristic polynomial of M is thus given by $F(X) = X^2 + [\gamma + e(g) - 2]X + [1 - e(g) - \gamma]$, and the roots of the polynomial equation $F(X)$ are 1 and $[1 - e(g) - \gamma]$.

The unit root indicates that we do not have a unique equilibrium steady-state values of $E^2(g)$ and $P^2(g)$, as we note in the text. However we do have a unique equilibrium solution for $S^2(g)$. Since $S_t^2 = E_t^2 P_t^2$, we obtain by adding the solutions for the rates of change in E_t^2 and P_t^2 in equation (B7) around the growth steady state

$$\hat{E}_{t+1} + \hat{P}_{t+1} = [1 - e(g) - \gamma]\left(\hat{E}_t + \hat{P}_t\right) \quad \text{or} \quad \hat{S}_{t+1} = [1 - e(g) - \gamma]\, \hat{S}_t.$$

Hence, we have stable and unique equilibrium for S_t as long as $-1 < [1 -$

$e(g) - \gamma] < 1$, or

$$-\gamma < e(g) < 2 - \gamma. \tag{B8}$$

3. Numerical Sensitivity Analysis of the Conditions for Local Stability

To perform sensitivity analyses of our local stability conditions, we evaluate the elasticity $e(s)$ by totally differentiating equation (B4) and then applying the baseline parameters used to derive the SE as well as the GE in table 1, parts 1 and 2. Equations (B4) and (B8) are found to be satisfied for all values of $\gamma \in (0, 1)$.

Appendix C

TABLE C1
Variables Used in the Regressions and Summary Statistics

Variable	Description	Mean (SD)
SD-FERT	Standard deviation of the distribution of surviving children per female ≥ 40	2.520 (.306)
AV-FERT	Average of the distribution of surviving children per female ≥ 40	4.179 (1.161)
SD-SCHYR	Standard deviation of the distribution of schooling years in the population ≥ 15	3.684 (.806)
AV-SCHYR	Average of the distribution of schooling years in the population ≥ 15	4.888 (2.755)
GINI*	Gini coefficient	37.76 (7.948)
QUINT*	Share of total income received by the top relative to the bottom quintile of families in the population	8.826 (5.259)
RGDPn	Real per capita income	6,340 (5,960)
GOV	GDP shares of government spending	19.53 (8.821)
INFLA	Annual inflation rate in GDP deflator (%)	56.39 (587.1)

* We calculate GINI and QUINT exclusively based on household income data reported in Dollar and Kraay (2001), excluding observations based on personal income, personal expenditures, or household expenditure data.

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