Social Interaction Models

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Glossary

Complementarity: A property of individual preferences or payoffs in which higher levels of some choice variable by others implies that higher levels of the choice variable are relatively more attractive to the individual.

Contextual effects: Effects determined by characteristics of an agent’s neighborhood.

Endogenous effects: Effects determined by neighborhood members’ contemporaneous behavior.

Neighborhood: Defined abstractly to be any group of individuals which could be considered to have a definable impact on an individual.

Peer group effects: An influence on an agent’s behavior due to the connection to or perception of a peer group.

Phase transition: A model exhibits phase transition if its properties qualitatively change for a small change in a parameter value. Phase transitions are thus a way of describing when threshold effects occur in an environment.

Poverty traps: A situation in which the incentives for increased income or wealth are offset by other effects such that a group of agents remain poor over long time periods.

Role model effect: An influence on an agent’s behavior due to the connection to or perception of a role model.

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Social multiplier: The notion that connections and interactions between individuals can amplify or reinforce direct influences on agents.

SOCIAL INTERACTIONS MODELS comprise a body of recent work by economists and other social scientists that attempts to analyze formally the interplay between individual decisions and social processes. Substantively, these models attempt to answer two broad classes of questions: First, how do the characteristics and choices of others affect an individual’s decisionmaking? Second, how are these social influences reflected in equilibrium behaviors observed in a group as a whole? Substantively, social interactions models extend the domain of economic reasoning by evaluating direct interdependences between individuals. As such, these models complement the traditional economic focus on individual interdependences that are mediated via prices. Social interactions refer to these direct interdependences. To understand the distinction, consider the determinants of teenage smoking. One reason why individual teenage smoking decisions are interdependent is that the choices of others will affect the price of cigarettes faced by an individual. In contrast, if smoking decisions are influenced by a desire to imitate one’s peers or one’s role models, then social interaction effects are present.

Over the past 15 years, there has been a renaissance of interest among economists in the social determinants of individual behavior and aggregate outcomes. A key reason for this is the potential for social interactions to help explain outstanding social questions such as the prevalence of inner-city poverty. In this regard, there are now a large body of empirical studies that attempt to measure social interaction effects on individuals in the context of residential neighborhoods; surveys of this work include Jencks and Mayer (1990) and Durlauf (2003). Social interactions models place such studies in a firm theoretical context. As well, methodological advances have allowed the incorporation of such effects into traditional microeconomic models. The basic structure and implications of these models suggest that these tools may have general application across the social sciences. Drawing from this basis, the summary here presents a basic overview of social interactions models and their applications, including a discussion of econometric techniques and outstanding research questions.

I. Social Interactions – Theory
By describing how agents' choices depend on the actions and characteristics of others in a common neighborhood, this work provides a basis for understanding how social factors combine with the market-based and individual-specific factors that are the basis of neoclassical economic reasoning.

Following Manski (1993), who adopts this terminology from the sociology literature, one can think of an agent's interactions with her neighborhood as being composed of two factors: contextual and endogenous. The first (contextual) refers to those factors that are group specific and based on characteristics of the group members. The second (endogenous) refers to how agents are affected by the contemporaneous behavioral choices of group members. These alternative factors are illustrated in the context of residential neighborhoods, which represent an important leading case in the social interactions literature. How do residential neighborhoods affect individual educational outcomes? One source is local public finance of schools. Such a mechanism links individual educational quality to the distribution of socioeconomic status among neighborhood families. Another mechanism is role model effects. In this case, a student's school effort may be influenced by the level of economic success he observes among adults in the neighborhood. Feedbacks from the socioeconomic status of adults in a community to student behavior is an example of a contextual effect. In general, contextual effects have the feature that they are not reflexive: while a student is affected by the behaviors of adult role models, he does not influence those role models per se. This is most obviously the case when the student is affected by a past behavior of an adult.

In contrast, endogenous effects refer to direct interdependences in contemporary choices among members of a neighborhood. For example, one might argue that the educational effort of one student is influenced by the effort of his friends; this type of endogenous effect is also known as a peer group effect. Unlike contextual effects, endogenous effects such as peer effects are reflexive; one student's effort influences his friends just as he is influenced. This is what is meant by an endogenous effect. Notice that both contextual and endogenous effects are influences that are not directly adjudicated by prices, i.e. there is no market to compensate an adult for providing a good role model or a student for providing desirable peer effects.
Policymakers have a particular interest in social interactions research in general, and endogenous effects in particular, in that they provide a mechanism for understanding two prominent issues in social science: poverty traps and social multipliers. To understand what is meant by poverty traps, suppose that the college attendance decision is strongly related to the percentage of graduates in the community. These connections in behaviors can lead to two communities with different levels of college graduates in the long-run. The mechanism for this should be clear: high (low) attendance rates of one generation lead to the high (low) rates for the next generation. Communities initially comprised of poor (via lack of education) members will remain poor across time. This result can be explained with intertemporal social interactions (i.e. social interactions in which choices made at one time affect others in the future).

Another conception of poverty traps comes from peer group effects. When such effects are strong, the characteristics of individuals in the group are not unique determinants of the group’s action; instead, dependence on history, reactions to common influences, etc. may determine which sort of average behavior actually transpires. The emphasis here is that strong contemporaneous dependences in behavior can generate multiple different self-reinforcing behaviors in groups. Within a given configuration of behaviors, each individual is acting “rationally” in the usual sense. Note that this does not suggest that each self-consistent configuration is equally desirable from the perspective of the members of the group. One can also interpret poverty traps as a socially undesirable collection of behaviors that are mutually reinforcing and consequently individually rational.

Social multipliers arise because social interactions can amplify the effects of individual incentives. For a policymaker, this means that alterations of private incentives across a group may have far larger per capita effects than that associated with one individual in isolation. Consider the impact of providing tertiary education scholarships to randomly chosen students across various high schools versus concentrating the funds amongst students within a given school. If the goal is to alter high school graduation rates, then the presence of social interactions can, other things equal, mean that the concentration of the scholarships will be more efficacious. With the assumption that the direct incentive effect of the scholarships is equal for all students, the advantage of
concentrating the scholarships in one school is that they will induce neighborhood effects for all students in the school, including those who have not been offered scholarships. Spreading the scholarships would have essentially no impact on any of the “neighborhoods” and would consequently impact only the students who received the funds. More generally, neighborhood effects can amplify the effects of altering private incentives; this amplification is what is meant by a social multiplier; see Glaeser, Sacerdote, and Scheinkman (1996) for detailed discussion. To date, the implications for policy design of such multipliers have been little explored.

The basic implications of these effects suggest that the notion of interactions within neighborhoods may have general application in varied social science contexts. Various research agendas now focus on populations of agents organized into groups in which some type of non-price related interactions occur. Each of these utilizes, at least abstractly, the notion of neighborhood-specific social interactions. Applications range from economic growth and development to crime to land use patterns. This work does not require that neighborhoods be defined geographically, but does rely on some notion of proximity versus distance in “social space,” a notion originally given content by Akerlof (1997).

II. Formal Theory

This section summarizes the above concepts into a formal model. First consider the abstract problem of how social interactions influence individual choices and thereby produce interesting neighborhood behaviors in the aggregate.

This model has \( I \) individuals part of a common neighborhood denoted \( n \). Each individual \( i \), chooses \( \omega_i \) from a set of possible behaviors \( \Omega_i \). This individual-level decision will produce a probabilistic description of the choice given certain features of the individual and his neighborhood. This model constructs a probability measure \( \mu(\cdot) \) for the vector of choices of all members of the group, \( \omega \), that is consistent with these individual-level probability measures and relates how neighborhood effects determine its
properties. To capture how others influence each agent, define $\omega_{-i}$ as the vector of choices made by individuals other than agent $i$.

Continuing from the theoretical discussion above, one may distinguish between the various types of influences on individual behavior. Adding to the contextual and endogenous factors defined above, one can also see two types of individual-specific characteristics: deterministic and random. These four types of influences have implications on how to model the choice problem. For simplicity, the model labels the four as follows:

$X_i$, a vector of deterministic (to the modeler) individual-specific characteristics associated with individual $i$,

$\epsilon_i$, a vector of random individual-specific characteristics associated with $i$,

$Y_n$, a vector of predetermined neighborhood-specific characteristics (these measure the contextual effects), and

$\mu^e_i(\omega_{-i})$, the subjective beliefs individual $i$ possesses about behaviors of others in her neighborhood, described as a probability measure over those behaviors (this term captures potential endogenous effects).

Each of these components will be treated as a distinct argument in the payoff function that determines individual choices. As discussed, the social interactions terms are the final two. Even though these may be "nonstandard" in the context of traditional economic decision problems in that they are not price-driven, individual choices are still defined via the maximization of some individual payoff function $V(\cdot)$; given the notation introduced, individual choices are thus assumed to follow:

$$\omega_i = \arg \max_{\omega_{-i}} V(\omega_i, X_i, \epsilon_i, Y_n, \mu^e_i(\omega_{-i}))$$

(1)

Next, to close this model, one must choose a method of resolving a standard problem in economics: how individuals form beliefs about the behaviors of others. The benchmark assumption in this literature is that beliefs are rational in the sense that

$$\mu^e_i(\omega_{-i}) = \mu(\omega_{-i} | \epsilon_i, Y_n, X_j, \mu^e_j(\omega_{-j}) \forall j)$$

(2)
where $j$ in this context refers to members of the neighborhood $n$ other than agent $i$. Notice that the right hand side of (2) is a conditional probability measure that describes how agent $i$ would form beliefs that are mathematically consistent with the model, given the conditioning variables. In particular, one should note that the distinction between one's beliefs about the choices of others and their actual choices here derives exclusively from the fact that agent $i$ observes only his own random payoff term, $\omega_i$. Finding an equilibrium set of behaviors in a neighborhood is thus a fixed-point problem, i.e. determining what subjective conditional probabilities concerning the behavior of others correspond to the conditional probabilities produced by the model when behaviors are based on those subjective beliefs.

To make the model more tractable for analysis and interpretation, equation (1) is often simplified in two ways. First, since one might expect individuals to care about the average behavior of those in the group, endogenous effects can be expressed as $\bar{\omega}_i = (I - \varepsilon_i)^{-1} \sum_{j \neq i} \omega_j$. Second, one can also remove uncertainty by setting $\varepsilon_i$ to zero for all neighborhood members; this allows expectations and realizations to coincide in (2). When these assumptions are made, individual decisions solve

$$\omega_i = \text{arg max}_{\omega \in \Omega} V(\omega_i, X_i, Y_n, \bar{\omega}_i)$$

(3)

From the perspective of formal theory, the interesting properties of social interactions models depend on the direct interdependencies that exist between individual choices, i.e. the endogenous effects that are captured by the presence of $\mu^\varepsilon(\omega_i)$ in (1) and $\bar{\omega}_i$ in (3). Social interactions models typically assume that these interdependencies between individual choices exhibit complementarity. Intuitively, complementarity means that the relative payoff of a higher value of $\omega_i$ versus a lower value is increasing in the levels chosen by others. For the payoff function described in (3), complementarity means that if $\omega_i^{\text{low}} < \omega_i^{\text{high}}$ and $\bar{\omega}_i^{\text{low}} < \bar{\omega}_i^{\text{high}}$, then

$$V(\omega_i^{\text{high}}, X_i, Y_n, \bar{\omega}_i^{\text{high}}) - V(\omega_i^{\text{low}}, X_i, Y_n, \bar{\omega}_i^{\text{high}}) > V(\omega_i^{\text{high}}, X_i, Y_n, \bar{\omega}_i^{\text{low}}) - V(\omega_i^{\text{low}}, X_i, Y_n, \bar{\omega}_i^{\text{low}})$$

(4)
Complementarity is a fundamental property for interdependent decisionmaking because it leads to similarity of behaviors as high choice levels by others make it more likely that an individual does the same; similar logic applies to low choice levels. In fact, the model (3) is an example of the class of coordination models that arise in applied game theory (e.g. Cooper and John (1988)).

What sorts of properties may be exhibited by social interactions models? One important property is that of multiple equilibria. A model such as (1)-(3) exhibits multiple equilibria when there is more than one set of choices \( \omega \) such that each individual is making the choice that maximizes his payoff. Intuitively, when complementarities are strong enough it permits individuals to behave similarly in equilibrium but does not specify or require particular behavior. This introduces a “degree of freedom” in the determination of outcomes as a whole. In the social interactions context, this is important because multiple equilibria create the possibility that two neighborhoods with similar observable characteristics (i.e. distributions of \( X_i \) within each neighborhood \( n \) and levels of \( Y_n \)) can exhibit different aggregate behaviors. When will multiple equilibria occur? Clearly, one factor is the strength of the endogenous social effects. If these effects are weak, then the other determinants of individual behavior play a relatively larger role in determining individual outcomes and can lead to unique equilibria.

A second property these models may exhibit is phase transition. A model exhibits phase transition if its properties qualitatively change for a small change in a parameter value. Phase transitions are thus a way of describing when threshold effects occur in an environment. Why do phase transitions occur in social interactions models? Intuitively, phase transition is related to the multiplicity versus uniqueness of equilibria. Social interaction models often have the property that, for a given specification of individual and contextual effects, there is a threshold for the strength of endogenous social effects such that if the level of endogenous effects is above the threshold, multiple equilibria occur.

Brock and Durlauf (2001a,b) provide an explicit analysis of how the strength of different factors that affect individual behavior jointly determine the number of equilibrium behaviors that may be observed at the group level for binary choice models with social interactions which illustrate these properties. In their framework, individuals
choose \( \omega_i \in \{-1, 1\} \). Social interactions are determined by the expected average choice level in the group, \( m_n \). Specifically, the payoff function is such that

\[
V(1, X_i, Y_n, \varepsilon_i) - V(-1, X_i, Y_n, \varepsilon_i) = k + cX_i + dY_n + Jm_n + \varepsilon_i
\]

(5)

where \( k, c, d, J \) are constants and \( \varepsilon_i \) is a scalar that is independently and logistically distributed, i.e. \( F_\varepsilon(z) = \frac{1}{1 + \exp(-z)} \). These papers show that the equilibrium expected average choice for a neighborhood must fulfill

\[
m_n = \int \tanh(k + cX + dY_n + Jm_n) \, dF_X
\]

(6)

where \( \tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)} \) and \( dF_X \) is the distribution of \( X \) within \( n \). The number of equilibrium values of \( m_n \) that is consistent with (6) is determined by the value of \( J \), holding other factors constant. For example, if each member of \( n \) is associated with the same individual effects, i.e. \( X_i = \bar{X} \), then the following results holds: For each value of \( k + c\bar{X} + dY_n \), there exists a threshold \( J^{\text{thresh}} \) (which depends on \( k + c\bar{X} + dY_n \)) such that if \( J < J^{\text{thresh}} \), then there is only one equilibrium whereas if \( J > J^{\text{thresh}} \), then three different values of \( m_n \) are consistent with (6).

To date, far less effort has been dedicated to analysis of econometric issues in social interactions topics than to theoretical work. General treatments include Brock and Durlauf (2001b, 2003), Manski (1993) and Moffitt (2001). These studies provide a number of important results for conducting and interpreting empirical work. This section highlights the causal mechanisms at work in the theoretical model we have described. Section III.A discusses identification issues, and III.B self-selection.

### III.A Identification

To provide an illustration of the basic identification issues in social interactions research, consider that \( V(\omega_i, X_i, \varepsilon_i, Y_n, \mu_i(\omega_{-i})) \) from above produces a linear representation of individual choice, i.e. choices obey the following basic regression specification
\[ \omega_i = k + cX_i + dY_{n(i)} + Jm_{n(i)} + \epsilon_i \]  

(7)

where as above, \( X_i \) denotes an \( r \)-length vector of observable individual characteristics, \( Y_{n(i)} \) denotes an \( s \)-length vector of contextual effects and \( m_{n(i)} \) denotes the expected value of \( \omega_i \) for members of neighborhood \( n(i) \) and \( n(i) \) denotes the neighborhood of individual \( i \) (which allows (7) to describe individuals from different neighborhoods). This model, referred to as the linear-in-means model, was first studied by Manski (1993). The linearity assumption facilitates interpretation as in a linear model. To put this in the context of the discussion above, note that all endogenous effects here work solely through expectations: an assumption that might be appropriate if the neighborhoods were particularly large. To highlight one of the key econometrics issues, we first focus on the case where \( E(\epsilon_i \mid X_i, Y_{n(i)}, i \in n(i)) = 0 \), such that identification questions are intrinsic to neighborhood effects rather than the endogeneity of the neighborhoods themselves.

To understand why identification conditions arise in this model, observe that when beliefs are rational,

\[ m_{n(i)} = \frac{k + cX_{n(i)} + dY_{n(i)}}{1 - J}. \]  

(8)

In this expression, \( X_{n(i)} \) equals the average of the \( X_i \)’s in neighborhood \( n(i) \) and appears in the regression because this average is one of the determinants of \( m_{n(i)} \). Substituting (8) into (7), the individual choices may be expressed in terms of observables via

\[ \omega_i = \frac{k}{1 - J} + cX_i + \frac{J}{1 - J}cX_{n(i)} + \frac{d}{1 - J}Y_{n(i)} + \epsilon_i \]  

(9)

Equation (9) summarizes the empirical implications of the linear-in-means model. The identification problem may thus be thought of as asking whether one can recover the structural parameters in (7) from the coefficients in (9).

To complete this analysis, one need only compare the number of regressors of (9) with the number of coefficients of (7). One can see that (9) contains \( 2r + s + 1 \) regressors while there are only \( r + s + 2 \) coefficients in (7). While it might appear that one could recover the structural parameters from a regression of \( \omega_i \) onto the various regressors, in fact the parameters of (7) are over-identified. However, this conclusion fails to account
for possible collinearity between the components of (9); collinearity may potentially arise because of the presence of $X_{n(i)}$ and $Y_{n(i)}$ in the equation. For example, following the case originally studied in Manski (1993), suppose that $X_{n(i)} = Y_{n(i)}$. In this case, the researcher would be unable to distinguish between contextual and individual effects. When this condition holds and there are only $r + s + 1$ linearly independent regressors in (9), the associated coefficients for these linearly independent regressors are identified, but they cannot be uniquely mapped back into the $r + s + 2$ structural coefficients in (7); identification of the structural parameters in (7) thus fails. Manski (1993) has termed this failure of identification the reflection problem, to capture the intuition that the identification problem relates to distinguishing the direct effect of $Y_{n(i)}$ on an individual versus its indirect effect as “reflected” through the endogenous effect generated by $m_{n(i)}$.

When does the identification problem preclude identification of the parameters of (7)? The key requirement for identification, cf. Brock and Durlauf (2001a,b), is that the vector $X_{n(i)}$ is linearly independent of the other regressors in (7), $\{1, X_{i}, Y_{n(i)}\}$. For this to be so, a necessary condition is that there exists at least one element of $X_{i}$ whose group level average does not appear in $Y_{n(i)}$. Intuitively, one needs prior information that at least certain individual level effects are present whose group level analogs do not affect individuals.

It is important to recognize that the identification problem discussed is a product of the linear specification. Identification breaks down when $m_{n(i)}$ is linearly dependent on the other regressors in (7); linear dependence of this type will typically not arise when individual behaviors depend on other moments of the neighborhood behavior. More important for empirical work, this argument also implies that identification will hold for nonlinear probability models of choices. For example, the binary choice model of Brock and Durlauf (2001a,b) is identified under weak assumptions.

III.B Self-Selection

The discussion of identification has not addressed the issue of self-selection into groups. For contexts such as residential neighborhoods, self-selection is of course
important. In fact, recent theories of neighborhood composition are driven by the presence of social interactions. From the perspective of our discussion, self-selection implies that $E(\varepsilon_i | X_i, Y_{n(i)}, i \in n(i)) \neq 0$.

There does not yet exist any general solution to the analysis of social interactions with self-selection. One approach, followed by Evans, Oates, and Schwab (1992) is to use instrumental variables to account for $E(\varepsilon_i | X_i, Y_{n(i)}, i \in n(i)) \neq 0$. An alternative approach, developed in Brock and Durlauf (2001b, 2003), models the self-selection explicitly. We focus on the linear-in-means model. Consider the following equation to illustrate the effect of self-selection on identification:

$$\omega_i = k + cX_i + dY_{n(i)} + Jm_{n(i)} + E(\varepsilon_i | X_i, Y_{n(i)}, i \in n(i)) + \xi_i$$

(10)

where $E(\varepsilon_i | X_i, Y_{n(i)}, i \in n(i)) = 0$ by construction. Following the classic approach to selection developed by James Heckman, e.g. Heckman (1979), consistent estimation of (10) requires constructing a consistent estimate that is proportional to $E(\varepsilon_i | X_i, Y_{n(i)}, i \in n(i))$, call it $\delta(X_i, Y_{n(i)}, i \in n(i))$ and including this estimate as an additional regressor in (10): i.e. one in essence estimates the regression

$$\omega_i = k + cX_i + dY_{n(i)} + Jm_{n(i)} + e\delta(X_i, Y_{n(i)}, i \in n(i)) + \xi_i$$

(11)

The key insight of Heckman is that once this is done, (11) may be estimated by ordinary least squares. Brock and Durlauf (2003) describe how to implement this procedure in the social interactions case using two-stage methods.

Self-selection corrections turn out to have important implications for identification. To see this, consider two cases. First, suppose that the decision to join a neighborhood depends only upon $m_{n(i)}$, i.e. $\delta(X_i, Y_{n(i)}, i \in n(i)) = \delta(m_{n(i)})$. In this case (11) is now nonlinear in $m_{n(i)}$ (since $\delta(\cdot)$ is almost certainly nonlinear given the fact that the neighborhood choice decision is made among a set of discrete alternatives) and is thus identified outside of pathological cases. Alternatively, in general $\delta(X_i, Y_{n(i)})$ will be linearly independent of $(1, X_i, Y_{n(i)})$ since $\delta(\cdot)$ is nonlinear. As such $\delta(X_i, Y_{n(i)})$ is an
additional individual level regressor whose group level analog does not appear in the behavioral equation (7). Thus identification may be achieved.

This approach to self-selection may be criticized to the extent that the self-selection correction is constructed on the basis of parametric assumptions concerning the distribution of the various model errors. We regard this as a legitimate but not critical caveat. The analysis in Brock and Durlauf (2003) which we have described should be interpreted as demonstrating that self-selection not only does not make identification of social interactions impossible, but may, if appropriately modeled, facilitate identification. This facilitation follows from the fact that neighborhood choices embody information on how individuals assess social interactions.

An equally important new direction is the development of data sets that will facilitate more detailed analyses of social interactions. One important development is the Moving to Opportunity Demonstration being conducted by the Department of Housing and Urban Development which involves creating incentives for poor families to move to more affluent neighborhoods in order to see how they are affected; valuable evidence on social interaction effects is found in Katz, Kling, and Liebman (2001). Other efforts are promising in terms of the detailed data that are being obtained. The Project on Human Development in Chicago Neighborhoods is noteworthy for the detailed information on attitudes and outcomes that is being compiled. The value of these data is well illustrated in Sampson, Morenoff and Earls (1999).
Bibliography


