Debt, Deficits, and Finite Horizons

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Many issues in macroeconomics, such as the level of the steady-state interest rate or the dynamic effects of government deficits, depend crucially on the horizon of agents. This paper develops a simple analytical model in which such issues can be examined and in which the horizon of agents is a parameter that can be chosen arbitrarily. The first part characterizes the dynamics and steady state of the economy in the absence of a government, focusing on the effects of the horizon index on the economy. The paper clarifies the separate roles of finite horizons and declining labor income through life in the determination of steady-state interest rates. The second part studies the effects and the role of fiscal policy. It clarifies the respective roles of government spending, deficits, and debt in the determination of interest rates.

This paper characterizes the dynamic behavior of an economy where agents have finite horizons. It then analyzes the effects and the role of government debt and deficits. There is in general no simple aggregate consumption function in an economy composed of finitely lived agents. This is because agents differ in two respects. Being of different ages, they have different levels and compositions of wealth. Having different horizons, they have different propensities to consume out of wealth. This systematic relation among wealth level, wealth composition, and propensity to consume makes exact or approximate aggregation impossible (Modigliani 1966).

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In view of this problem, the solution adopted by Diamond (1965) was to choose a very simple population and age structure, avoiding altogether the need for aggregation. The solution chosen in this paper is, instead, to make assumptions that allow aggregation. The central assumption is that agents face, throughout their life, a constant instantaneous probability of death p. Thus their expected life is $(1/p)$; furthermore, it is constant throughout their life. Agents are of different ages and have different levels of wealth, but have the same horizon and the same propensity to consume. This allows one to solve the aggregation problem.

The main advantage of this approach is its flexibility. If we think of $(1/p)$ as the horizon index, we can choose it anywhere between zero and infinity and study the effects of the horizon of agents on the behavior of the economy. In particular, by letting p go to zero, we obtain the infinite horizon case as a limiting case.

The main drawback of this approach is that it captures the finite horizon aspect of life but not the change in behavior over life, the "life-cycle" aspect of life. In that respect it is closer to the initial formulation of permanent income by Friedman (1957) than to that of life cycle by Modigliani (1966). It is well adapted to issues where the finite horizon aspect is important, such as issues of debt and deficits. It is poorly adapted to issues where differences in propensity to consume across agents are potentially important.

Section I derives the behavior of both individual and aggregate consumption. The aggregate consumption has a particularly simple and tractable form: aggregate consumption is a linear function of aggregate financial and human wealth. Human wealth is the present discounted value of labor income accruing in the future to those currently alive.

Sections II and III characterize the behavior of an economy of agents with finite horizons. Section II studies the dynamic behavior and steady states of both open and closed economies. Section III considers two extensions. The first focuses on the effects of the elasticity of substitution of consumption. The second allows for declining labor income through life, to capture the effects of the "saving for retirement" motive on capital accumulation. The effect of finite horizons, per se, is to decrease capital accumulation. The effect of declining labor income is, however, to increase it. The net effect is ambiguous, and the resulting steady state may well, as in Diamond, be inefficient.

Section IV introduces the government. Because the focus is on intertemporal reallocations of taxes, the government is assumed to have lump-sum taxes at its disposal. The section introduces the government budget constraint and shows how finite horizons imply a role

for debt policy. It then derives an "index of fiscal policy" that summarizes the effects of current and anticipated fiscal policy on aggregate demand. This index has two components. The first reflects the effects of government spending and shows how both the level and expected changes in spending affect aggregate demand. The second captures the effects of government finance and shows how both the level of debt and the expected sequence of deficits affect aggregate demand. The importance of this second component is smaller the longer the horizon of agents and disappears when agents have infinite horizons. Section V shows the steady-state effects of fiscal policy in the open and closed economies; Section VI characterizes its dynamic effects by considering two examples. The first, inspired by the current fiscal policy in the United States, is that of a reallocation that creates high deficits followed later by surpluses. The second, which leads to the study of optimal debt policy, studies the role of debt policy in smoothing aggregate consumption in the face of regular fluctuations in output.

I. The Aggregate Consumption Function

The derivation of aggregate consumption is based on two major assumptions. The first specifies the probability of death and the structure of population: Time is continuous. Each agent throughout his life faces a constant probability of death p.[1] At any instant of time, a large cohort, whose size is normalized to be p, is born.[2]

If the probability of death is constant, the expected remaining life for an agent of any age is given by $\int_0^\infty tpe^{-pt}dt = p^{-1}$. I shall refer to p^{-1} as the horizon index. As p goes to zero, p^{-1} goes to infinity: agents have infinite horizons.

The assumption that cohorts are large implies that, although each agent is uncertain about the time of death, the size of a cohort declines nonstochastically through time. A cohort born at time zero has a size, as of time t, of pe^{-pt}, and the size of the population at any time t is $\int_{-\infty}^{t} pe^{-p(t-s)}ds = 1$.

[1] An alternative interpretation, suggested by Robert Barro, is to think not of agents but of families. Then p is the probability that either the family ends—i.e., that members of the family die without children—or the current members of the family have no bequest motive. The assumption of a constant p is more acceptable under this interpretation. How unrealistic is the assumption of a constant p? Evidence on mortality rates suggests low and approximately constant probabilities from age 20 to age 40. After this, mortality rates are well summarized by "Gomperty's Law" (see Wetterstrand 1981): $g_y = 1 - e^{-\mu_y}$; $\mu_y = BC^y$, where y is age, g is the mortality rate, B and C are positive constants. Estimates are, e.g., $g_{50} = 1$ percent; $g_{60} = 3$ percent; $g_{80} = 16$ percent; $g_{100} = 67$ percent.

[2] I assume for simplicity that there is no population growth. Introducing population growth in the form of larger new cohorts over time is straightforward.

In the absence of insurance, uncertainty about death implies that agents may leave unanticipated bequests although they have no bequest motive. They may also be constrained to maintain a positive wealth position if they are prohibited from leaving debt to their heirs. Under my assumptions, private markets may, however, provide insurance risklessly, and it is reasonable and convenient to assume that they do so. This motivates the second assumption:[3] There exist life insurance companies. Agents may contract to make (or receive) a payment contingent on their death.

Because of the large number of identical agents, such contracts may be offered risklessly by life insurance companies. Given free entry and a zero profit condition, and given a probability of death p, agents will pay (receive) a rate p to receive (pay) one good contingent on their death.

In the absence of a bequest motive, and if negative bequests are prohibited, agents will contract to have all of their wealth (positive or negative) return to the life insurance company contingent on their death. Thus, if their wealth is w, they will receive pw if they do not die and pay w if they die.

These two assumptions are sufficient to characterize aggregate consumption. For simplicity, I shall, however, make two further assumptions. The first, which implies a simple individual consumption function, is that utility is logarithmic. The second, which implies a simple form for aggregate human wealth, is that labor income is distributed equally across agents. I shall relax these two assumptions in Section III.

Individual Consumption

Denote by $c(s, t)$, $y(s, t)$, $w(s, t)$, $h(s, t)$ consumption, noninterest income, nonhuman wealth, and human wealth of an agent born at time s, as of time t. Let $r(t)$ be the interest rate at time t. Under the assumption that instantaneous utility is logarithmic, the agent maximizes[4]

$$E_t\left[\int_t^{\infty} \log c(s, v)e^{\theta(t-v)}dv\right], \quad \theta \geq 0.$$

[3] The role of insurance when there is uncertainty about time of death was studied by Yaari (1965). An equivalent assumption is that there exist actuarial bonds. Lenders lend to intermediaries. These claims are canceled by the death of the lenders. Borrowers borrow from intermediaries; these claims are canceled by the death of the borrowers. Intermediation can again be done risklessly.

[4] The assumption of a constant probability of death implies that the objective function (eq. [1]) does not change through time. There is therefore no issue of time inconsistency of initial optimal programs.

Given the constant probability of death p, and if the only source of uncertainty is about the time of death, maximizing the above is equivalent to maximizing

$$\int_t^\infty \log c(s, v)e^{(\theta+p)(t-v)}dv. \tag{1}$$

The effective discount rate is therefore $(\theta + p)$. Even if θ is equal to zero, agents will discount the future if p is positive.

If an agent has wealth $w(s, t)$ at time t, he receives $r(t)w(s, t)$ in interest and $pw(s, t)$ from the insurance company. Thus its dynamic budget constraint is

$$\frac{dw(s, t)}{dt} = [r(t) + p]w(s, t) + y(s, t) - c(s, t). \tag{2}$$

An additional transversality condition is needed to prevent agents from going infinitely into debt and protecting themselves by buying life insurance. I impose a condition that is the extension of that used in the deterministic case.[5] The solution must be such that if the agent is still alive at time v

$$\lim_{v\to\infty} e^{-\int_t^v [r(\mu)+p]d\mu} w(s, v) = 0.$$

If this is the case, the budget constraint can be integrated to give

$$\int_t^\infty c(s, v)e^{-\int_t^v [r(\mu)+p]d\mu}dv = w(s, t) + h(s, t), \tag{3}$$

where

$$h(s, t) = \int_t^\infty y(s, v)e^{-\int_t^v [r(\mu)+p]d\mu}dv.$$

The agent maximizes (1) subject to (3). This problem is very similar to the deterministic case, except for the presence of $(\theta + p)$ and $(r + p)$ instead of θ and r. As utility is logarithmic, the solution is simply

$$c(s, t) = (p + \theta)[w(s, t) + h(s, t)]. \tag{4}$$

Individual consumption depends on total individual wealth, with propensity $(\theta + p)$. The discount rate used to discount labor income is $(r + p)$, the same as the rate at which nonhuman wealth accumulates.

[5] This condition is the extension to infinite time of the condition proposed by Yaari (1965). Throughout the paper, when characterizing the behavior of consumption given r and y, I assume that, at least asymptotically, r is larger than $-p$. This rules out pathological cases.

Aggregate Consumption

Denote aggregate variables by uppercase letters. The relation between any aggregate variable $X(t)$ and an individual counterpart $x(s, t)$ is

$$X(t) = \int_{-\infty}^{t} x(s, t)pe^{p(s-t)}ds.$$

Let $C(t)$, $Y(t)$, $W(t)$, $H(t)$ denote aggregate consumption, noninterest income, nonhuman wealth, and human wealth at time t, respectively. Then from equation (4), aggregate consumption is given by

$$C(t) = (p + \theta)[H(t) + W(t)].$$

Aggregate consumption is a linear function of aggregate human and nonhuman wealth. The next step is to characterize the dynamics of both components of aggregate wealth.

Human wealth is given by

$$\begin{aligned} H(t) &= \int_{-\infty}^{t} h(s, t)pe^{p(s-t)}ds \\ &= \int_{-\infty}^{t} \left\{ \int_{t}^{\infty} y(s, v)e^{-\int_t^v [r(\mu)+p]d\mu} dv \right\} pe^{p(s-t)}ds. \end{aligned}$$

Changing the order of integration gives

$$H(t) = \int_{t}^{\infty} \left[\int_{-\infty}^{t} y(s, v)pe^{p(s-v)}ds \right] e^{-\int_t^v r(\mu)d\mu} dv.$$

This has a simple interpretation. The term in parentheses is labor income accruing at time v to agents already alive at time t. Human wealth is thus the present value of future labor income accruing to those currently alive. To characterize the dynamic behavior of $H(t)$, we need to specify the distribution of labor income across agents. We shall assume for the moment that labor income is equally distributed (i.e., that all agents work and have the same productivity): $y(s, v) = Y(v)$ for all s. Thus, all agents have the same human wealth and $H(t)$ is given by

$$H(t) = \int_{t}^{\infty} Y(v)e^{-\int_t^v [r(\mu)+p]d\mu} dv$$

or, in differential equation form,

$$\frac{dH(t)}{dt} = [r(t) + p]H(t) - Y(t)$$

$$\lim_{v \to \infty} H(v)e^{-\int_t^v [r(\mu)+p]d\mu} = 0.$$

Nonhuman wealth is given by

$$W(t) = \int_{-\infty}^{t} w(s, t) p e^{p(s-t)} ds.$$

Differentiating with respect to time gives

$$\frac{dW(t)}{dt} = w(t, t) - pW(t) + \int_{-\infty}^{t} \frac{dw(s, t)}{dt} p e^{p(s-t)} ds.$$

The first term on the right is the financial wealth of newly born agents, which is equal to zero. The second term is the wealth of those who die. The third is the change in the wealth of those alive. Using equation (2) gives

$$\frac{dW(t)}{dt} = r(t)W(t) + Y(t) - C(t).$$

Whereas individual wealth accumulates, for those alive, at rate $r + p$, aggregate wealth accumulates at rate r. This is because the amount pW is a transfer, through life insurance companies, from those who die to those who remain alive; it is not therefore an addition to aggregate wealth.

Collecting equations, dropping the time index, denoting $d\cdot/dt$ by a dot gives a first characterization of aggregate consumption:

$$C = (p + \theta)(H + W) \tag{5}$$

$$\dot{H} = (r + p)H - Y \tag{6}$$

$$\dot{W} = rW + Y - C. \tag{7}$$

If agents have finite horizons, if $p > 0$, the discount rate on noninterest income in (6) exceeds the interest rate.[6]

There is an alternative characterization of the behavior of aggregate consumption that will be useful. Differentiating (5) and eliminating $\dot{H}$ and $\dot{W}$ gives

$$\dot{C} = (r - \theta)C - p(p + \theta)W; \tag{8}$$

$$\dot{W} = rW + Y - C. \tag{9}$$

If agents have infinite horizons, $p = 0$ and equation (8) reduces to the standard equation (e.g., Hall 1978). If $p > 0$, the rate of change of C depends also on nonhuman wealth. Note that even if p is positive, individual consumption follows $\dot{c} = (r - \theta)c$. Thus if $r = \theta$, individual

[6] Such a specification, allowing for a higher discount rate for human wealth, has been estimated by Hayashi (1982). His estimated coefficients, α, μ, and ρ, are related to p, θ, and r by $p = \mu - \rho$; $\theta = \alpha - \mu + \rho$; $r = \rho$. His estimates (table 1, $\lambda = 0$) imply at annual rates $p = .10$; $r = .03$; $\theta = -.03$.

consumption will be constant but aggregate consumption will in general vary.

II. Dynamics and Steady State

I consider in turn the cases of a closed and an open economy.

The Open Economy

In the open economy, the interest rate is the world interest rate, r, which is given and at which consumers can freely borrow and lend. For simplicity, there is no capital and the only assets are therefore the net holdings of foreign assets, denoted F. Noninterest income is exogenous and denoted ω. Using equations (8) and (9), equilibrium is characterized in this case by

$$\dot{C} = (r - \theta)C - p(p + \theta)F; \tag{10}$$

$$\dot{F} = rF + \omega - C. \tag{11}$$

Note that one can also think of this system as giving the partial equilibrium dynamics of consumption, savings, and wealth, given r and ω, in a closed economy.

This system is linear in C and F. It is saddlepoint stable if r is less than $\theta + p$. If r were larger than $\theta + p$, individual consumption would increase at a rate larger than p, that is, larger than the rate of death; aggregate consumption would therefore increase forever. If we exclude this case, we have a well-defined steady state, with associated values for C and F. This result differs sharply from the infinite horizon case where a steady-state value of C exists only if $r = \theta$ and where in this case the value of F is indeterminate (or more precisely, depends on the path of adjustment).

I now construct the phase diagram. The slope of $\dot{C} = 0$ is positive if $r > \theta$, negative if $r < \theta$. The slope of $\dot{F} = 0$ is positive. Both cases are represented in figures 1*a* and 1*b*. In both cases, the stable arm is upward sloping.

What are the characteristics of this steady state? If $r = \theta$, the value of F is zero. Agents have flat labor income and consumption profiles; they do not save or dissave. If r is greater than θ, individual consumption is increasing, agents are accumulating over their life, and the level of foreign assets is positive. If r is smaller than θ, agents are decumulating and, as a result, the level of foreign assets is negative. The country is a net debtor in steady state.

An increase in r increases the level of foreign assets. An increase in

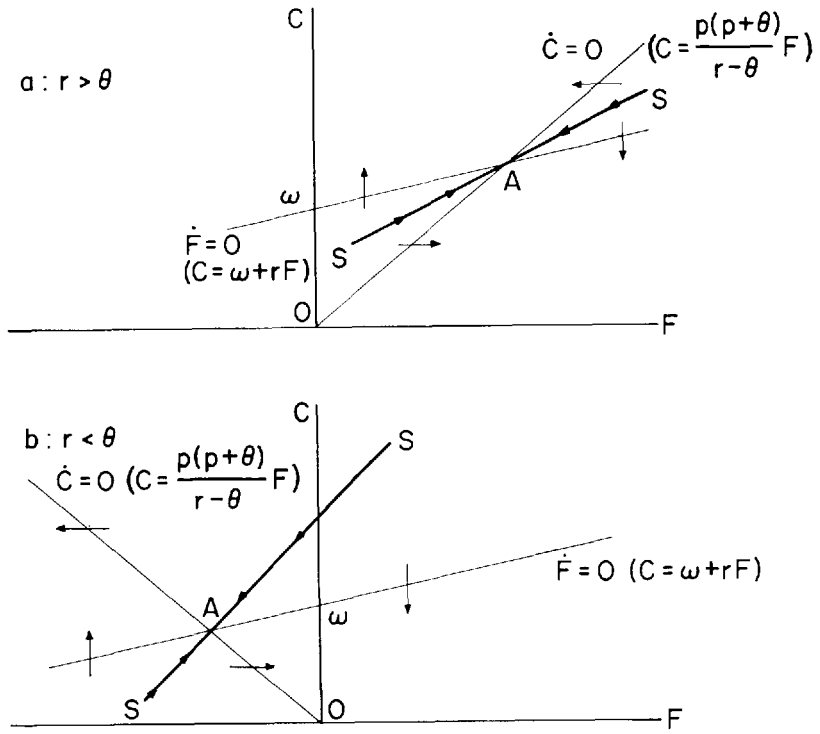

FIG. 1

p pushes the level of foreign assets toward zero, reducing it if positive and increasing it if negative: shorter horizons imply smaller aggregate accumulation or decumulation.

An alternative way of looking at aggregate behavior is to return to the specification giving aggregate consumption as a function of wealth and to derive an aggregate savings function. $C = (p + \theta)[\omega/(r + p) + F]$, whereas income is $\omega + rF$, so that $S \equiv Y - C = [(r - \theta)/(r + p)]\omega + (r - p - \theta)F$. As r is less than $\theta + p$, savings is a decreasing function of wealth F.[7] The effect of noninterest income ω is ambiguous and depends on $(r - \theta)$. In steady state, savings must be equal to zero. If r is equal to θ, this implies a zero equilibrium level of foreign assets. If r is greater than θ, the equilibrium level is positive; if r is less than θ, the equilibrium level is negative.

[7] This savings function is similar to that used by Dornbusch and Fischer (1980) in their study of current account dynamics.

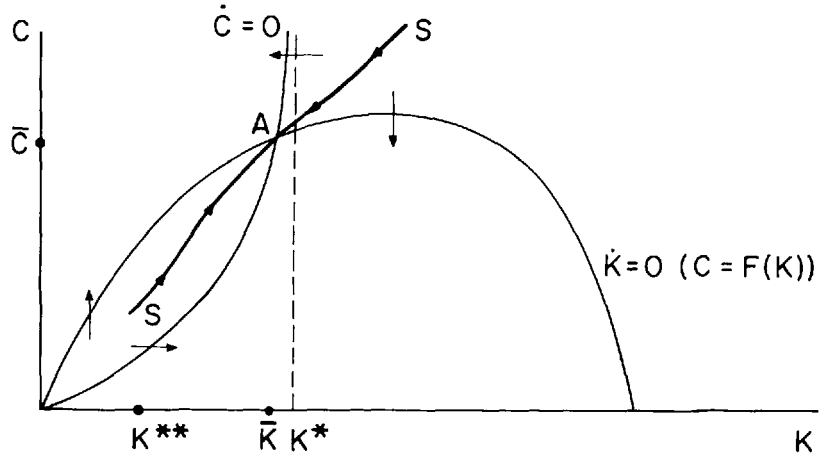

FIG. 2

The Closed Economy

In the closed economy, ω and r are no longer given but are determined instead by capital accumulation. There are two factors of production, capital K and labor; from above, the size of the population is equal to the size of the labor force and equal to unity. Let $\tilde{F}(K, 1)$ be the constant returns to scale production function and δ be the depreciation rate. Define $F(K) \equiv \tilde{F}(K, 1) - \delta k$.

Nonhuman wealth is equal to K. Noninterest income is labor income $\omega(K)$. The interest rate is the net marginal product of capital, $r(K)$, which may be positive or negative. Using equations (8) and (9) gives

$$\dot{C} = [r(K) - \theta]C - p(p + \theta)K \tag{12}$$

$$\dot{K} = F(K) - C. \tag{13}$$

Figure 2 characterizes the phase diagram associated with the system. Let us define two values of K, K^* such that $r(K^*) = \theta$, and K^{**} such that $r(K^{**}) = \theta + p$. The locus $\dot{C} = 0$ is upward sloping, going through the origin and asymptotically reaching K^*. The locus $\dot{K} = 0$ traces the production function.

The equilibrium is unique with a saddlepoint structure. The stable arm SS is upward sloping. Any other trajectory can be shown to imply a negative level of C or K in finite time, and thus the stable arm is the only acceptable trajectory: given K, C is uniquely determined.

The steady-state interest $\bar{r}$ is between θ and $\theta + p$: $\bar{r} \geq \theta$ follows from $K < K^*$; $\bar{r} < \theta + p$ is shown by contradiction. Suppose $\bar{r} \geq \theta + p$

so that $(\bar{r} - \theta)\bar{C} \geq p\bar{C}$. From equation (12) and $\dot{C} = 0$, $(\bar{r} - \theta)\bar{C} = p(p + \theta)\bar{K}$, so that $(p + \theta)\bar{K} \geq \bar{C}$. From equation (13) and $\dot{K} = 0$, $\bar{C} = F(\bar{K})$, so that $(p + \theta)\bar{K} \geq F(\bar{K})$. As by assumption $\bar{r} \geq \theta + p$, $\bar{r}\bar{K} > F(\bar{K})$, which is impossible.

If $p = 0$, so that agents have infinite horizons, $\bar{r} = \theta$: the standard "modified golden rule" result obtains. When p is positive, however, $\bar{r}$ is larger than θ. The reason is clear if we return to the individual consumption equation $\dot{c} = (r - \theta)c$. In a steady state, labor income is constant through life. In order to generate positive aggregate capital, agents must be saving initially and consumption must be increasing. Thus $\bar{r}$ must be larger than θ.

Furthermore, $\bar{r}$ is an increasing function of p. This follows from the phase diagram: an increase in p shifts $\dot{C} = 0$ to the left, decreasing $\bar{K}$. The shorter the horizon, and the higher the interest rate, the lower the level of steady-state capital. I shall show, however, in the next section that declining labor income during life has the opposite effect.

III. Two Extensions

I consider now two extensions to the original model. The first focuses on the effects of the elasticity of substitution of consumption, the second on the effects of declining labor income through life.

The Role of the Elasticity of Substitution

I now consider the class of isoelastic utility functions:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma \neq 1,$$

$$= \log c, \quad \sigma = 1.$$

The elasticity of substitution is given by σ^{-1}. All other assumptions are unchanged.

Following the same steps as in Section I, the first-order condition to the individual maximization problem is

$$\frac{dc(s, t)}{dt} = \sigma^{-1}[r(t) - \theta]c(s, t). \tag{14}$$

Solving for $c(s, v)$, $v \geq t$ as a function of $c(s, t)$, replacing in the budget constraint given by equation (3), and solving for $c(s, t)$ gives

$$c(s, t) = [\Delta(t)]^{-1}\,[w(s, t) + h(s, t)]$$

$$\Delta(t) = \int_t^{\infty} e^{\sigma^{-1}\int_t^v\{(1-\sigma)[r(\mu)+p]-(\theta+p)\}\,d\mu}\,dv.$$

As before, consumption is a linear function of total wealth. The propensity to consume is now a function of the sequence of future interest rates; it is, however, not a function of age and is therefore the same for all agents.

Following the same steps as in Section I gives the dynamic behavior of aggregate consumption:

$$C = \Delta^{-1}(H + W)$$

$$\dot{\Delta} = -1 - \sigma^{-1}[(1 - \sigma)(r + p) - (\theta + p)]\Delta,$$

or, equivalently,

$$\dot{C} = \sigma^{-1}(r - \theta)C - p\Delta^{-1}W$$

$$\dot{\Delta} = -1 - \sigma^{-1}[(1 - \sigma)(r + p) - (\theta + p)]\Delta,$$

where the behavior of Δ is characterized by a differential equation (and a transversality condition I have not written) and the dynamic behavior of H and W is the same as in Section I.

I shall limit my analysis to the effects of σ on steady-state capital in a closed economy. The steady state is characterized by

$$\dot{C} = \sigma^{-1}[r(K) - \theta]C - p\Delta^{-1}K = 0 \tag{15}$$

$$\dot{K} = F(K) - C = 0 \tag{16}$$

$$\dot{\Delta} = -1 - \sigma^{-1}\{(1 - \sigma)[r(K) + p] - (\theta + p)\}\Delta = 0. \tag{17}$$

Figure 3 characterizes the steady state graphically. The first locus is $\dot{C} = \dot{\Delta} = 0$, or

$$C = pK\frac{\{(\sigma - 1)[r(K) + p] + (\theta + p)\}}{r(K) - \theta}.$$

For $\sigma > 1$, this locus starts at the origin, is upward sloping, and approaches K^* asymptotically, where K^* is again such that $r(K^*) = \theta$. For $\sigma < 1$, the locus is initially downward sloping, then upward sloping. It also approaches K^* asymptotically. These two cases are drawn in figures 3*a* and 3*b*. The second locus, $\dot{K} = 0$, traces the net production function.

The steady-state capital stock is smaller than K^*; the net marginal product $\bar{r}$ is thus greater than θ. An increase in σ shifts the $\dot{C} = \theta$. locus to the left, decreasing steady-state capital. Thus, the lower the elasticity of substitution, the lower steady-state capital. Intuition for this result is obtained by examining equation (14), giving the behavior of individual consumption. As in Section II, a positive aggregate capital requires savings initially in life. As labor income is flat, this requires initially low and increasing consumption. The lower σ^{-1}, the larger the interest rate needed to twist the consumption path. Equiva-

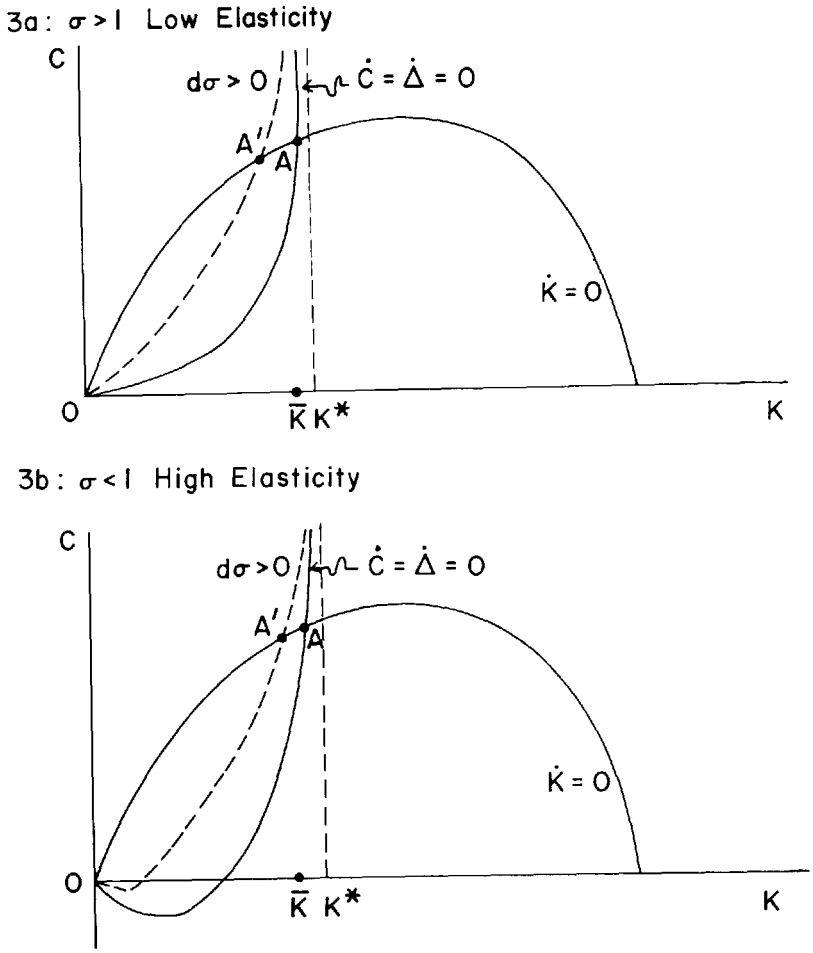

FIG. 3

lently, the lower σ^{-1}, the lower initial individual savings given the interest rate.

The Effects of Declining Labor Income

What I want to capture here are the effects of "saving for retirement" on aggregate capital accumulation. Introducing retirement, that is, zero labor income after some given length of life, is not analytically convenient. I assume instead that labor income declines with age at rate α. More precisely, I assume

$$y(s, v) = aY(v)e^{\alpha(s-v)}, \tag{18}$$

where a is a constant to be determined. The share of labor income received by an agent is an exponentially decreasing function of age.[8] The value of a is determined by the condition that

$$\begin{aligned} Y(v) &= \int_{-\infty}^{v} y(s, v) p e^{p(s-v)} ds \\ &= aY(v) \int_{-\infty}^{v} p e^{(p+\alpha)(s-v)} ds = \left(\frac{pa}{p+\alpha}\right) Y(v) \\ \Rightarrow a &= \frac{p+\alpha}{p}. \end{aligned}$$

Note that the case where α is positive is well defined only if p is strictly positive. If $p = 0$, agents are infinitely long lived. Individual labor income must be the same, up to a constant, as aggregate labor income; it cannot be a decreasing function of age.

The derivation of individual consumption, assuming logarithmic utility, is identical to that of Section I. The derivation of aggregate consumption is also the same, except for aggregate human wealth. Given (18), individual human wealth is given by

$$h(s, t) = e^{\alpha(s-t)} \left\{ \left(\frac{p+\alpha}{p}\right) \int_{t}^{\infty} Y(v) e^{-\int_t^v [\alpha + r(\mu) - p] d\mu} dv \right\}.$$

Note that the term in braces is the same for all agents. Thus aggregate human wealth is

$$\begin{aligned} H(t) &\equiv \int_{-\infty}^{t} h(s, t) p e^{p(s-t)} ds \\ &= \int_{t}^{\infty} Y(v) e^{-\int_t^v [\alpha + r(\mu) + p] d\mu} dv. \end{aligned}$$

The effect of declining labor income is to increase the discount rate on future aggregate labor income. This is because agents currently alive will receive, even if still alive in the future, a smaller and smaller share of total income.

Collecting equations gives the following:

$$C = (p + \theta)(H + W) \tag{19}$$

$$\dot{H} = (r + p + \alpha)H - Y \tag{20}$$

$$\dot{W} = rW + Y - C. \tag{21}$$

[8] This formalization can be extended to accommodate more complex paths of income. If one wants to capture the fact that labor income initially increases and then decreases with age, this can be done by assuming that the share is the sum of two negative exponentials, i.e., is equal to $a_1 e^{\alpha_1(s-v)} + a_2 e^{\alpha_2(s-v)}$, $\alpha_1, \alpha_2, a_1 < 0, a_2 > 0, a_1\alpha_1 + a_2\alpha_2 > 0$.

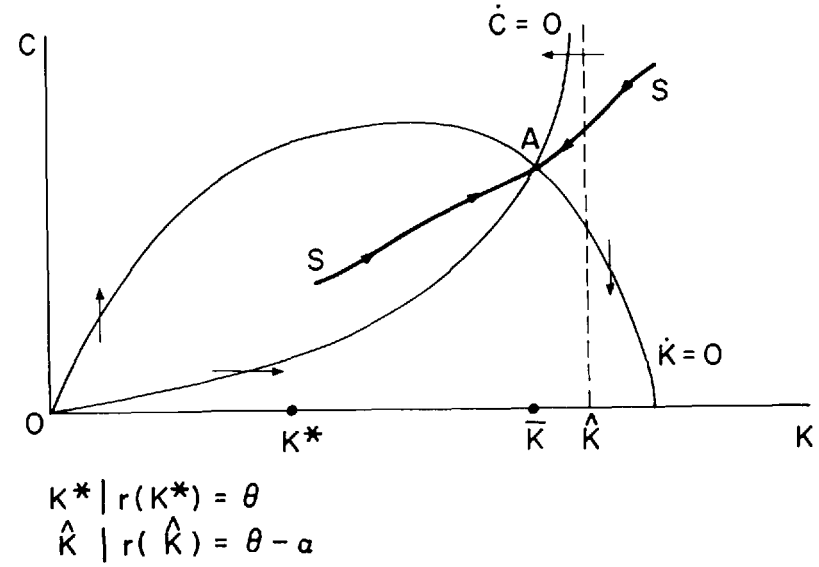

FIG. 4

The equation for human wealth is given in differential form. The only effect of α is to increase further the discount rate on future labor income above the interest rate.

The alternative characterization of the dynamics, obtained by differentiating (14) and eliminating $\dot{H}$ and $\dot{W}$ using (20) and (21), is

$$\dot{C} = (r + \alpha - \theta)C - (p + \alpha)(p + \theta)W,$$

$$\dot{W} = rW + Y - C.$$

I now turn to the dynamics and steady state of the closed economy. The dynamics are characterized by

$$\dot{C} = [r(K) + \alpha - \theta]C - (p + \alpha)(p + \theta)K, \tag{22}$$

$$\dot{K} = F(K) - C. \tag{23}$$

The phase diagram associated with (22) and (23) is drawn in figure 4. The $\dot{C} = 0$ locus goes through the origin, is convex, and reaches $\hat{K}$ asymptotically, where $r(\hat{K}) = \theta - \alpha$. Note that $r(\hat{K})$ may be positive or negative: figure 4 is drawn so that $r(\hat{K})$ is negative. The $\dot{K} = 0$ locus traces the net production function. The equilibrium is saddlepoint stable. The stable arm SS is upward sloping.

What are the characteristics of the steady state? The interest rate $\bar{r}$ is smaller than $\theta + p$, and larger than $\theta - \alpha$.

The proposition that $\bar{r}$ is larger than $\theta - \alpha$ follows from $\bar{K} < \hat{K}$.

That $\bar{r}$ is less than $\theta + p$ is again proven by contradiction. Suppose that $\bar{r} \geq \theta + p$. Then $(\bar{r} - \theta + \alpha)\bar{C} \geq (p + \alpha)\bar{C}$. Equation (22) and $\dot{C} = 0$ imply $(\bar{r} - \theta + \alpha)\bar{C} = (p + \alpha)(p + \theta)\bar{K}$, so that $(p + \theta)\bar{K} \geq \bar{C}$. Equation (23) and $\dot{K} = 0$ imply $\bar{C} = F(\bar{K})$, so that $(p + \theta)\bar{K} \geq F(\bar{K})$. As by assumption $\bar{r} \geq \theta + p$, we get $\bar{r}\bar{K} \geq F(\bar{K})$, which is impossible.

It is quite possible for $\bar{r}$ to be negative. As the golden rule in this case is $r = 0$—there is no population growth—the capital stock may exceed the golden rule and the economy may be, as in Diamond, dynamically inefficient.

Furthermore, an increase in α—that is, a more sharply declining labor income path—increases steady-state capital and decreases $\bar{r}$. To see this, consider $dC/d\alpha|_{\dot{C}=0,\, K=\bar{K}} = \bar{K}(p + \theta)(\bar{r} - \theta - p)/(\bar{r} - \theta + \alpha)$. As $\bar{r} < \theta + p$ and $\bar{r} > \theta - \alpha$, $dC/d\alpha|_{\dot{C}=0,\, K=\bar{K}} < 0$. In the neighborhood of steady state, an increase in α shifts the $\dot{C} = 0$ locus to the right, increasing steady-state capital. The reason for this result is clear: if labor income accrues relatively early in life, it will lead to more savings early in life and thus higher aggregate wealth.

To summarize, the effect of finite horizons per se is to decrease capital accumulation. The effect of declining labor income on saving for retirement is to increase it, however. The net effect is ambiguous and the steady state can be inefficient.

IV. Effects of Taxes on Aggregate Demand

The Government Budget Constraint

I now introduce a government that spends on goods that do not affect the marginal utility of private consumption and finances spending either by lump-sum taxes or by debt.[9] Its dynamic budget constraint is $\dot{D} = rD + G - T$, where D is debt, G is spending, and T is taxes. I shall refer to $T - G$ as the surplus, or deficit as the case may be, and to $\dot{D}$ as the change in debt. This is only a semantic convention. The government is also required to satisfy the transversality condition:[10]

$$\lim_{t \to \infty} D_t e^{-\int_t^v r_v dv} = 0.$$

This condition, together with the dynamic budget constraint, is equivalent to the statement that the level of debt is equal to the present discounted value of future surpluses:

$$D_t + \int_t^\infty G_s e^{-\int_t^v r_v dv} ds = \int_t^\infty T_s e^{-\int_t^v r_v dv} ds. \tag{24}$$

[9] The assumption that the government can use lump-sum taxation is made to focus on the effects of finite horizons. As is well understood, if lump-sum taxation is not available, there is a role for debt policy even if agents have infinite horizons.

[10] In what follows, I assume that r is nonnegative, at least asymptotically. I therefore do not look at fiscal policy in the case of dynamic inefficiency.

The presence of taxes modifies slightly the aggregate consumption function of Section I, which becomes

$$C_t = (p + \theta)(H_t + W_t); \; W_t = D_t + K_t \tag{25}$$

$$H_t = \int_t^{\infty} Y_s e^{-\int_t^s (r_v + p)dv} ds - \int_t^{\infty} T_s e^{-\int_t^s (r_v + p)dv} ds \tag{26}$$

$$\dot{W}_t = rW_t + Y_t + C_t - T_t. \tag{27}$$

Financial wealth, W, now includes government debt, D, and other assets, K. Human wealth is the present discounted value of noninterest income minus taxes, discounted at rate $(r + p)$.

Effects of a Reallocation of Taxes

Consider a decrease in taxes at time t associated with an increase at time $t + \tau$. Given the government budget constraint (24), the level of debt D_t, and an unchanged path of G, these changes must satisfy

$$dT_{t+\tau} = -e^{+\int_t^{t+\tau} r_v dv} dT_t.$$

The effect on the consumer at time t is given by the effect on human wealth. Equation (26) implies an effect of

$$-dT_t - dT_{t+\tau} e^{-\int_t^{t+\tau} (r_v + p)dv}$$

or, using the government budget constraint, an effect of $-dT_t(1 - e^{-p\tau})$. Thus, unless $p = 0$, a decrease in taxes today increases human wealth and consumption. The longer taxes are deferred, the larger the effect. This effect of a reallocation of taxes comes from the different discount rates in the government budget constraint and in the definition of human wealth. This in turn reflects the fact that taxes are partly shifted to future generations: $(1 - e^{-p\tau})$ is simply the probability that someone currently alive will not have to pay the future increase in taxes.

An Index of Fiscal Stance

Fiscal policy—that is, the sequence of current and anticipated taxes, spending, and debt—affects aggregate demand in three ways. Debt is part of wealth and affects consumption; the sequence of taxes affects human wealth and thus consumption. The level of government spending affects aggregate demand directly. It is useful, both conceptually and technically, to summarize these effects by an index of fiscal policy. Let g denote this index, so that, collecting all the terms in aggregate demand that depend directly on fiscal policy,

$$g_t = (p + \theta)\left[D_t - \int_t^{\infty} T_s e^{-\int_t^s (r_v + p)dv} ds\right] + G_t.$$

This index can be rewritten as

$$g_t = G_t - (p + \theta)\int_t^{\infty} G_s e^{-\int_t^s (r_v + p)\,dv} ds + (p + \theta)\left[D_t + \int_t^{\infty} (G_s - T_s) e^{-\int_t^s (r_v + p)\,dv} ds\right]. \tag{28}$$

The first line gives the effects of spending on aggregate demand if it is financed exclusively by contemporaneous taxes. Effects of spending are not the subject of this paper. Note, however, that if $r = \theta$, a constant level of spending has no effect on aggregate demand. The second line gives the effects of financing. Note first that if $p = 0$, the government budget constraint (24) implies that this line is identically equal to zero; financing is irrelevant. If p is positive, this is not the case, and if D_t is positive, this term is likely to be positive. The second line in effect measures the degree to which debt is net wealth and the degree to which it is offset by anticipated future surpluses. It makes clear that the degree to which debt is wealth depends on the whole sequence of anticipated surpluses (deficits).

How does this index evolve over time for a given fiscal policy? Consider a policy characterized by large deficits initially, followed later, as debt accumulates, by surpluses so as to satisfy the intertemporal government budget constraint. Does the initial increase in g disappear as surpluses appear, or does g remain positive as debt accumulates? An example, inspired by the current U.S. fiscal policy, will suggest the answer. The specific fiscal policy we consider is characterized by

$$\dot{D}_t = rD_t - T_t;\ T_t = \beta D_t - Z;\ D_0 = 0. \tag{29}$$

Government spending is equal to zero for simplicity. Taxes are an increasing function of the level of debt. A necessary and sufficient condition for the government transversality condition to be satisfied is that $\beta \geq r$, so that an increase in debt reduces $\dot{D}$. The interest rate r is assumed constant.

Consider the effect of an increase at time t_0 of Z from zero, which implies initially a sequence of deficits. As debt accumulates, taxes increase, surpluses appear eventually, and debt converges to a new steady-state value, D_∞, where $D_\infty = Z/(\beta - r)$. The smaller $(\beta - r)$, the longer the sequence of deficits, and the higher the steady-state level of debt.

Solving (29) for the path of D and T and replacing in (28) gives

$$g_t = \left[\frac{(p + \theta)p}{\beta - r}\right][(r + p)^{-1} - (\beta + p)^{-1}e^{-(\beta - r)t}]Z \tag{30}$$

so that in particular

$$g_0 = \left[\frac{(p + \theta)p}{(r + p)(\beta + p)}\right]Z$$

$$g_\infty = \left[\frac{(p + \theta)p}{(r + p)(\beta - r)}\right]Z = \left[\frac{(p + \theta)p}{r + p}\right]D_\infty > g_0.$$

The value of g at time t_0, g_0, depends on the sequence of anticipated deficits and thus on β. The steady-state value of g, g_∞, is easy to understand. In steady state, taxes required to pay interest on the debt are equal to rD_∞. Their present value in human wealth is $-[r/(r + p)]D_\infty$, and thus the net effect of debt is $(p + \theta)\{D_\infty - [r/(r + p)]D_\infty\}$, which is equal to g_∞.

The index therefore increases from g_0 to g_∞: although the budget goes from deficit to surplus, the increase in debt dominates, leading to an increase in g.

I now turn to the general equilibrium effects of fiscal policy.

V. Steady-State Effects of Fiscal Policy

My focus in this section is on the steady-state effects of debt on the level of consumption and holdings of other assets. These effects are best understood by considering the dynamic effects of the following rather artificial policy experiment. Starting from steady state, the government, at some time t_0, issues and distributes new additional debt while increasing taxes to pay for the additional interest payments. The level of debt remains constant at its new level forever. If the interest rate changes over time, taxes are adjusted to always cover interest payments. I consider first the open economy and then the closed economy.

The Open Economy

In the presence of fiscal policy the equations of motion are for the open economy:

$$C = (p + \theta)\left(\frac{\omega - T}{r + p} + D + F\right)$$

$$\dot{F} = rF + \omega - C - G$$

$$\dot{D} = rD + G - T.$$

The fiscal policy I consider is such that D, G, and T are constant (and thus $\dot{D}$ is equal to zero), except at time t_0 when D and T increase permanently. Their increases satisfy $dT_0 = rdD_0$.

One can solve for steady-state $\bar{C}$ and $\bar{F}$ as functions of D and G (T is determined implicitly by the government budget constraint):

$$\bar{F} = (p + \theta - r)^{-1}(r + p)^{-1}[(r - \theta)(\omega - G) - (p + \theta)pD];$$

$$\bar{C} = \omega - G + r\bar{F}.$$

The steady-state level of foreign assets is a decreasing function of the level of government debt and so is steady-state consumption. More precisely $d\bar{F}/dD = -(p + \theta - r)^{-1}(r + p)^{-1}(p + \theta)p \gtreqless -1$ as $r \gtreqless \theta$.

Government debt displaces foreign assets in agents' wealth. The displacement is one for one if $r = \theta$ (note that in this case $\bar{F} = -D$), but it may be much larger if r is larger than θ. These results differ sharply from the infinite horizon case where the level of debt has no effect on the steady-state level of F. By choosing its level of debt, the government can choose any level of steady-state consumption it desires.

These effects are easier to understand if one examines the process of adjustment and the savings function. Consumption is given by $(p + \theta)[(\omega - T)/(r + p) + D + F]$ and income by $\omega - T + r(D + F)$, so that savings are given by

$$S = \left(\frac{r - \theta}{r + p}\right)(\omega - G) + (r - p - \theta)F - p\left(\frac{p + \theta}{r + p}\right)D.$$

Thus the change in fiscal policy at time t_0 implies a decrease in savings of $dS_0/dD_0 = -p(p + \theta)/(r + p)$.

The increase in taxes and debt does not affect income but leads agents to feel wealthier by an amount $[p/(r + p)]dD_0$. This leads them to increase consumption and dissave and to decumulate foreign assets. This decumulation proceeds until a lower level of foreign assets has been reached and savings are again equal to zero. By then, both foreign assets and consumption are lower.

The Closed Economy

The equations of motion are in this case

$$\dot{C} = [r(K) - \theta]C - p(p + \theta)(D + K);$$

$$\dot{K} = F(K) - C - G;$$

$$\dot{D} = r(K)D + G - T.$$

I consider the same fiscal policy as before, that is, an increase of D and T at time t_0 that satisfies $dT_0 = r(K_0)dD_0$. As the interest rate will

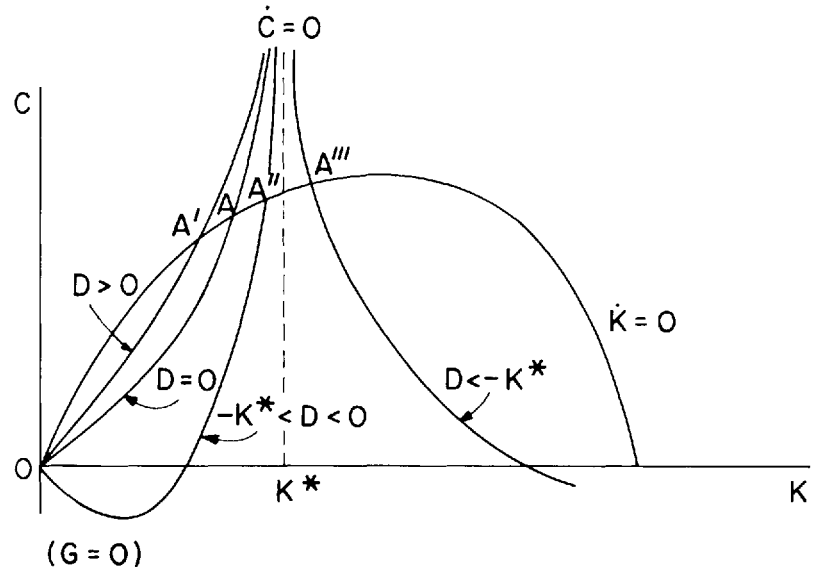

FIG. 5

now vary after time t_0, taxes implicitly vary after time t_0 to cover interest payments on the new constant level of debt.

I characterize the steady state graphically in figure 5. Let K^* be such that $r(K^*) = \theta$. As long as D is larger than $(-K^*)$ the $\dot{C} = 0$ locus goes through the origin and reaches K^* asymptotically from the left. If D is negative and less than $(-K^*)$, the $\dot{C} = 0$ locus reaches K^* asymptotically from the right. The $\dot{K} = 0$ locus is drawn for a value of G equal to zero.

If G equals zero and debt is larger than $(-K^*)$, there is a unique saddle-point stable equilibrium. If G is positive, there might be zero or two equilibria. The same is true if debt is less than $(-K^*)$. The cases of zero and two equilibria will not be considered further.

The effect of an increase in government debt is to shift the $\dot{C} = 0$ locus to the left and thus to decrease the steady-state levels of capital and consumption. The government can again choose any level of capital by an appropriate choice of debt. If, for example, the government uses $\theta + p$ as the social discount rate, it may achieve the desired level of capital K^{**}, such that $r(K^{**}) = \theta + p$, by issuing a positive amount of debt. If instead it uses θ as the social discount rate, it must issue a negative amount of debt, in this case precisely an amount equal to $-K^*$.

The dynamics of adjustment to the increase in debt are qualitatively

similar to those of the open economy. The increase in debt and taxes creates an initial wealth effect on consumption, leading to capital decumulation. In the new steady state, capital and consumption are lower.

I now turn to the dynamic effects of more realistic fiscal policies.

VI. Dynamic Effects of Specific Fiscal Policies

Internal and External Deficits

The first policy I consider is one that resembles the current U.S. policy, in which debt is initially low and deficits are large and in which, as debt accumulates, the government slowly moves from deficits to surpluses until debt has stabilized at a new higher level. I limit my analysis to the case of an open economy; the focus is therefore not on the interest rate, which is given, but on the relation between government and current account deficits.

The fiscal policy is the same as that described in Section IV and is implemented unanticipatedly at time t_0. It is convenient in this case to use the index of fiscal policy defined in Section IV and derived for this particular policy, also in Section IV. The equations of motion can be written as

$$C = (p + \theta)\left(\frac{\omega}{r + p} + F\right) + g;$$

$$\dot{F} = rF + \omega - C;$$

$$\dot{g} = (r - \beta)g + \left[\frac{(p + \theta)p}{r + p}\right]Z$$

$$g_0 = \left[\frac{(p + \theta)p}{(r + p)(\beta + p)}\right]Z.$$

As government spending is by assumption equal to zero, fiscal policy has an effect only through consumption. The behavior of g was characterized in equation (30) and is written here equivalently in differential equation form. At time t_0, g increases from zero to g_0.

The dynamics of F and g are characterized in figure 6. The stability condition is that r be less than $\theta + p$. If it is satisfied, the locus $\dot{F} = 0$ is downward sloping, with slope $(r - p - \theta)^{-1}$. The $\dot{g} = 0$ locus is vertical.

The steady state, prior to the change in fiscal policy, is point E. At t_0, the locus $\dot{g} = 0$ shifts to the right, and g jumps from 0 to g_0. The system jumps from E to A. Over time, the economy converges from A to E'.

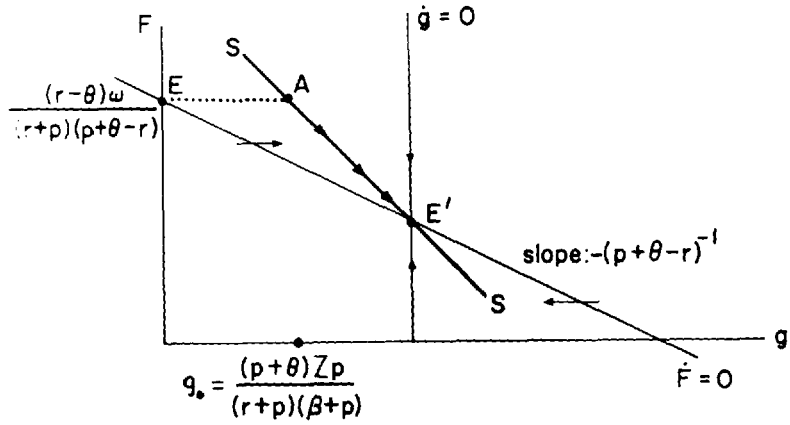

FIG. 6

The effect of fiscal policy is thus of a decumulation of foreign assets as government debt accumulates. The rates of foreign asset accumulation and government debt accumulation are not, however, related in any simple way: the rate of foreign asset decumulation depends not only on the current deficit but also on the entire sequence of deficits (surpluses). For example, if $r = \theta$, we have seen in the previous section that in the long run the decrease in foreign assets is equal to the increase in government debt. At $t = 0$, however, the effect of the rate of change of government debt on the rate of change in foreign assets is given by $d\dot{F}/d\dot{D}|_{t_0} = -g_0/Z = -p/(\beta + p)$. Thus the short-run effect is less than one for one. It tends to one as p tends to infinity, as the horizon of agents shortens. As β reaches its lower bound r (i.e., the lowest value consistent with satisfaction of the government transversality condition), it tends to $p/(r + p)$.

Output Cycles and the Role of Debt Policy

This second example examines the effects of regular fluctuations in output on the decentralized economy and the role of debt in such a case.

The economy is open, and for simplicity I assume $r = \theta$. If I define total financial wealth W as the sum of foreign assets and government debt, the equations of motion can be written as

$$\dot{C} = -p(p + \theta)W$$

$$\dot{W} = rW + \omega - C - T$$

$$(\dot{D} = rD + G - T).$$

Now suppose that ω_t follows: $\omega_t = \Psi + \sin t$; $\Psi > 0$. In the absence of fiscal policy ($T = D = G = 0$), what will the behavior of C and W be?

Some algebra yields

$$C_t = a_1 \sin t + a_2 \cos t + \Psi,$$

$$W_t = b_1 \sin t + b_2 \cos t,$$

where

$$\begin{aligned} a_1 &= \Delta p(p + \theta)[1 + p(p + \theta)] &&\doteqdot p(p + \theta) \\ a_2 &= -\Delta p(p + \theta)\theta &&\doteqdot p(p + \theta)\theta \\ b_1 &= \Delta\theta &&\doteqdot \theta \\ b_2 &= -\Delta[1 + p(p + \theta)] &&\doteqdot -1 \end{aligned}$$

and

$$\Delta \equiv \{\theta^2 + [1 + p(p + \theta)]^2\}^{-1}.$$

The approximations to a_1, a_2, b_1, b_2 hold if p and θ are not far from zero.

If $p = 0$, agents completely smooth out consumption. Aggregate consumption is constant. Agents accumulate foreign assets when ω is high and decumulate when ω is low.

If $p > 0$, aggregate consumption is cyclical. As $r = \theta$, each agent still has a flat consumption path. The newly born do not, however, have the same level of consumption as those who die. If $\theta = 0$, aggregate consumption moves in phase with income, but by less.

These movements in aggregate consumption suggest a role for fiscal policy. As $r = \theta$, the consumption of each individual is constant throughout life. Different cohorts, however, have different levels of consumption. Thus, if the social welfare function is concave in individual utilities, it is desirable to smooth consumption across cohorts. Fiscal policy can achieve constant aggregate consumption over time. The equations of motion above show how this can be done: T must simply equal $\sin t$. If all deviations of ω from its mean Ψ are taxed, consumption and wealth are constant. Government debt and foreign assets follow symmetric but opposite paths. As government debt increases, for example, it displaces foreign assets in the agents' portfolios one for one.

This change in portfolio composition has no effect, in an open economy, on either ω or r. This would not be the case if the same debt policy were pursued in a closed economy: variations in the capital stock would affect both ω and r. Characterization of optimal policy would be substantially more difficult.

VII. Conclusion

The purpose of this paper was to characterize rigorously the effects of intertemporal reallocations of taxes when agents have finite horizons. To that end, many assumptions, such as the existence of lump-sum taxes or a constant employment level, were made that need to be removed to obtain a more realistic characterization of the effects of debt and deficits. The aggregate consumption function developed here seems well adapted to the task. The index of fiscal policy should also prove useful both conceptually and empirically.

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