WHEN AND WHERE IS A CITY FRACTAL?[1]

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Abstract. The paper presents an analysis of the development of the Tel Aviv metropolis using the concept of fractal. The fractal density of the whole urban ensemble and of its parts was estimated as a function of the time, from 1935 on. The central part and the northern tier is fractal at all times. Its fractal dimension increase with time. However, the whole ensemble can be said to be fractal only after 1985. There is a general tendency toward fractality in the sense that the fractal dimensions of the different parts converge toward the same value.

INTRODUCTION

Ever since the concept of "fractal" was proposed by Mandelbrot [1983], it has been applied extensively to describe a variety of physical phenomena (see for example Fleischmann, et. al., [1989]). In recent years the concept has been applied in the context of human geography as well. In two recent books, by Batty and Longley [1994] and by

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Frankhauser [1994], the claim was made that urban morphology is fractal. A number of studies have reported estimates of the fractal dimension of several large cities. The estimated values reported are around 1.7, and sometimes larger (see Anas et.al. [1998]).

There are a number of issues concerning urban morphology that have not received sufficient attention. It is not self-evident, for example, that all cities are fractal. It is not self-evident that fractality is a general property of all cities. Should it become evident that some urban structures are not fractal, it will become interesting to classify cities according to the type of morphology that they exhibit. It will become interesting to explain why the development of some towns has a fractal character and why in some cases such characteristic is absent. Furthermore, Batty & Longley [1994] have reported that the fractal dimension, D, of London changes over time. A similar result was reported concerning Berlin by Frankhauser [1994]. These results suggest that it is quite likely that a city becomes fractal only at some stage of its development. The third issue concerns the limits, or boundaries, of urban phenomena. It is not easy to define them a priori and it is not clear whether assuming different boundaries would yield different estimates of fractal dimensions. As an example, one can consider the case of London. If one takes the values of D as reported by Batty and Longley, it is noteworthy that D does not increase regularly but exhibits an isolated maximum in 1840. This does not seem very reasonable. The source of this strange result may be in the choice of city limits. It is possible that different boundaries would have yielded a different, and perhaps more reasonable, result.

The study reported on herein is concerned with the above issues and in particular with the morphology of the Tel Aviv metropolis from 1935 to 1991. The purpose of the study was to determine the fractal dimension as the function of time for the whole urban ensemble and for different parts of it. The important conclusion of this work is that only the northern tier of the metropolis is fractal throughout its history, while the whole ensemble appears to have become fractal only recently. However, there is a clear tendency towards fractality of the entire ensemble. We conclude that the use of the concept of fractal must be with some caution and that a critical examination is necessary before deciding whether a city is fractal or not.

THE METHOD

The basis for our estimates of the fractality of the Tel Aviv metropolis is a series of maps of the Tel Aviv ensemble that were prepared by Portugali as part of a study of urban growth. The series of maps from 1935 to 1991 is displayed in figure 1.

To determine the fractal density of Tel Aviv the box counting method was used. The choice of method is important as it can affect the conclusion whether an object is fractal or not. According to this method, a grid made of squares, or cells, of size l covers the object under study. Then, the number of cells in which a part of the object appears is counted (see figure 2). The resulting number is a function of the exact size of cells, i.e. of l. A series of values of l were taken. At first the size of a square in which the entire object is contained was defined as L. Thus $l_1 = L$. The next value l_2 is equal to L/2. The ith value of l is $l_i = L /(2^i$

). In our case, the greatest value of the index i is 9. Since $2^9 = 512$, the value of L is chosen equal to 512 and $l_9 = 1$.

The maps of Tel Aviv were digitized by means of a scanner and with the help of a computer, the series of values of N_i, corresponding to the series of values of l_i was determined.

In the next step, the points ($\log N_i$, $\log l_i$) were plotted. In the ideal case of a fractal object, the result is a perfect straight line with a slope equal to –D. D is the resulting estimate of the fractal dimension (for demonstration of this point, see Vicsek [1989], and Batty and Longley [1994]).

In the ideal case, the fractal dimension is estimated by a regression fit of a straight line that best represents the distribution of the individual point estimates. Modern cities, however, have time-limited histories and thus a limited number observations is possible. To overcome the limited number of observations an alternative computerized fitting program was used. Always, it yields estimates $D \pm \delta$, where δ is an estimation error. To conclude whether an urban object is fractal indeed, it is imperative to choose the maximum value of acceptable δ. To build-in a greater sensitivity into the results, two extreme examples were plotted in figure 3. The figure presents results (Log N versus Log l) for the entire ensemble in 1952 and for the central part in 1991 (below, the limits of the central part are presented separately). It is evident that in the first case, the points are not on a straight line and the object is not fractal. This is despite the fact that the computerized result yields a value for the slope of the line in figure 3. But in the second case, the straight line is a very good approximation of the distribution of the point estimates and it can be concluded that the object is a fractal.

In the case of a paucity of observations there does not seem to exist a rigorous rule to define the largest acceptable value of the error, δ_m. But, even a casual visual inspection seems to suggest that a fair value for δ_m is 0.040. There is some arbitrariness in this visual inspection and in the resulting decision. Yet, it does not modify seriously the conclusions, inasmuch as in the extreme cases there does not seem to be any ambiguity as to the result. For some intermediate cases, however, the results may be less obvious, thus requiring some caution in their interpretation.

THE MAPS AND THE DIVISIONS

A geographic map of the Tel Aviv metropolis is presented in figure 4. The structure of the whole ensemble is visible clearly. The seacoast limits the development of the town on the west. The city of Tel Aviv can be seen at the center of the ensemble. More than twenty smaller towns are dispersed around the center.

The maps used for the estimation of the fractal dimension are shown in figure 1. The maps indicate in black the ground surface occupied by buildings only. Markings of land occupied by roads and streets have been removed.

Three regions were chosen (see figure 5). For each of these, the series of values of the quantities Log N_i, log l_i was determined. The values of D and of δ were estimated by means of the above-described fitting program. The first region is the central part (region 1). It contains Tel Aviv and the five towns that are contiguous to it (Givataim, Ramat Gan, Bnei Brak, Holon, Bat Yam). The common characteristic of these towns is that they were always urban centers, while the majority of the other towns in the ensemble began as agricultural

settlements. This is evident from the examination of the evolution of their population over time. For the towns belonging to the central part, the curves do not exhibit any anomaly at the time of the creation of the State of Israel (1948). Other towns display an upward discontinuity at this time.

The second region includes the central part and the northeast part of the entire ensemble. It was chosen because visually the built area in this region appears to be more developed than in the south.

Finally, the third region is the entire ensemble. It is at this geographic scale that the problem of the region's boundaries becomes apparent. On the east there is no issue as to the boundary. There is a "natural" boundary since urban expansion is limited by the frontier between Israel and the occupied territories. The boundaries in the north and in the south were chosen so that beyond the boundaries the main land use is agricultural. It is a rather loose definition. Yet, it should not cause an important error in the resulting conclusions.

THE RESULTS

Results for the three regions are presented in two forms: in table I and in figure 6. Inspection of the table suggests that region 1 is always fractal. It displays a small error, always smaller or equal to 0.025. As was already evident from inspection of figure 3, the quality of the fit for the curve Log N versus Log l is very good. Thus, without hesitation, it can be concluded that the central part of the Tel Aviv metropolis is fractal. It is also evident that over time D increases from 1.533 to 1.809 with a small discontinuity (or rapid increase) between the years 1971-1978.

Region 2 is fractal over the entire study period. This is because at all times the error is smaller that the maximum value we defined above. However, the values of D are much smaller that those of region 1 (central part). The rapid increase of D is clearly evident between the years 1941 and 1952, a period during which these settlements experienced an intensive population growth. The difference in the values of D for regions 1 and 2 decreased regularly. In 1935 the difference was 0.221 and in 1991 it became equal to 0.142.

For region 3, the entire ensemble, things are different. The error is relatively large and only after the year 1985, one can say that the Tel Aviv metropolis is fractal. To support this conclusion, we display in figure 7 the curve Log N versus Log l, for the year 1991. Effectively, a straight line is a good fit. Here also the anomaly between the years 1942 and 1952 is evident.

A comparison of the three curves presented in table I and in figure 6 suggests a tendency towards fractality. It can be observed that there is a general trend of increase in the values of D. Assuming a linear increase from 1978 on, it is evident that the slopes of the three regions are increasing, from region 3 to the region 1. From a linear extrapolation of the three curves, it seems that they will converge around the year 2010.

To conclude this section, a comparison between regions 1 and 3 is presented. Region 1, that was always an urban ensemble, is fractal with an increasing fractal dimension D. In 1991, D had a similar value to other large cities, such as Paris and London. Also, there is a rapid increase between 1971 and 1978, although there seems to have been no marked event during this period (but, more on this point below).

Region 3 exhibits an important change around 1950 when a large immigration waive took place immediately after the creation of the state in 1948. This is expressed by the rapid

increase of D during this period. Later on, a regular increase of D is observed. But, the ensemble becomes fractal only after 1985. It is a consequence of the choice of δ_m. Since δ decreases from 1971 on, one expects that after 1991 this decrease will continue. Therefore, even with a different choice of δ_m, it is likely that region 3 would become fractal, but at a somewhat later time.

DISCUSSION

The first and most important conclusion from the above is that regions 1 and 2 of the Tel Aviv urban ensemble display a spatial evolution that follows a fractal pattern.

Three periods in the development of the œnter are distinguishable: before 1940, between 1942 and 1971, and after 1971. Only one value of D was estimated for the first period. In the second and third periods, D increases linearly with a smaller slope in the third period. It is interesting to compare the evolution of the population during these periods (see figure 8). From 1950 to 1975 the population increases linearly and there is no sign of rapid variation in D values between 1971 and 1978. After 1975 and until 1990, the population of region 1 is practically constant while D increases slightly. One can conclude that this last increase in D is due to building for other land-uses, rather than for residential housing. In fact, it is well known that an important business center has been developing during the 1980s in the core of the metropolis. During the same time period many of its residents moved to various more remote suburban locations.

As far as the entire ensemble is concerned, the discontinuity in D between 1942 and 1952 indicates the birth of the metropolis as a large urban ensemble. Before the creation of

the state and the large immigration waves that followed it, the urban ensemble was in its central part only. Very shortly thereafter, numerous settlements were progressively transformed into secondary urban centers.

Since the north part is fractal, but the whole ensemble is not, it must be concluded that the south part of the metropolis was not fractal until 1985. This conclusion was verified directly. It is noteworthy that nothing special occurred in the Tel Aviv metropolis, nor in Israel, in 1985. Yet, the resulting finding suggests that the structure and the growth mode of the north and the south (excluding the central part) were completely different. In other words, the Tel Aviv metropolis is heterogeneous. Here then, the observer is confronted with an unexpected situation. On the one side, it must be concluded that the Tel Aviv metropolis is not fractal before 1985, implying different development patterns for different parts of the metropolis. But on the other hand, it can be concluded that before 1985 the south part is not an organic part of the metropolis. An intermediate point of view is possible as well. Accordingly, the south belongs to the ensemble and only progressively its fractal nature appears.

No matter which interpretation is adopted, the clear tendency of the metropolitan ensemble towards fractality is an interesting behavior. It is expected that in less that fifteen years the ensemble will have a uniform fractal dimension (around 1.8). It is entirely plausible that at that time the boundaries of the metropolis will be different. This, then, is a special feature of the urban growth: fractality appears progressively. In other words, it is probable that a large town is not always fractal but becomes fractal at some stage of its development. It should be very interesting to find the phenomena associated with apparition of fractality. This would also be important for the understanding of the concept itself.

The patterns of spatial evolution described above, namely the transformation of the urban structure of region 3 around the year 1985 from non-fractal to fractal and the rapid change of D of the central part between 1971 and 1978, require an explanation. Several aspects of regional growth and development were considered.

The rate of population growth of the entire region during the 1970s does not indicate any significant changes that might explain the essential change in values of the fractal dimension D. On the other hand, an examination of Israel's real per capita Gross Domestic Product (GDP) provides convincing evidence of rapid economic growth in the late 1960s and the early 1970s, as compared to the uniform and almost linear growth since 1950 and until today (figure 9). This change in growth patterns is attributed usually to the influence of the Six-Day War on the Israeli economy. The

increase in the national product resulted, among other things, in increased volume of business activity and in improvement in the population's standard of living.

One of the expected byproducts of economic prosperity was improvement in the housing stock of the population of big cities in the central part of the country. To achieve higher housing standards residents of cities migrated to locations with newer and more spacious housing. Generally, the intra-urban migration was away from the central city of the metropolis and toward suburban communities in a rural environment. Hence, following the rapid economic growth between 1967 and 1971 the population of the older urban centers spread out to the countryside. Generally, the new housing developments were around existing small towns within the Tel Aviv metropolis. This leap-frogging process (see Benguigui et. al. [1998]), increased the homogeneity of the built area and the extent to which the development filled the open space of the metropolis.

The spread of urban activities around main urban poles was responsible for the observed change in the fractal dimension in region 3. It is likely that this "1960/70s" effect first gave an impetus to the development of the business sector, and the associated expansion of commercial building stock in the central part of the metropolis, and only afterwards to the growth in housing stock. There seems to be no doubt that it gave rise to a discontinuity in D between 1971 and 1978 and to fractality of the whole ensemble.

Finally, and most importantly, there is no interpretation, or indeed a model, that provides an explanation of the increase in the fractal dimension as growth takes place (Vicsek, Mandelbrot). All the models of fractal growth maintain D constant. This can be understood very easily in the case of deterministic fractals, where a fractal is created by self-similarity (Mandelbrot). It seems that this is a particular property of town growth and this point necessitates further investigation.

CONCLUSIONS

From our study of the Tel Aviv metropolis two important conclusions are suggested. First, fractality appears in some areas of the urban ensemble at early stages of its development, such as in the central part and the north east region, and in some area it evolves progressively, as in the case of the entire ensemble. This finding suggests that in the same ensemble different parts have their own mode of development. The second finding is that the fractal dimension increases over time. This clearly means that the density also

increases with time. But, the structure of a fractal is conserved; i.e. it has the same structure at different scales. It is not clear why a town is indeed fractal. Important effort has to be made in order to relate fractality to the other properties of town growth. The tentative interpretation of the sudden variation of D of the central part between the years 1971 and 1978 and apparition of a fractal city after 1985 is suggestive of the type of explanations needed.

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